QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION

Jorge J. Martínez de Lejarza

Jorge.M.Lejarza@ific.uv.es

Based on: Jorge J. Martinez de Lejarza, Michele Grossi, Leandro Cieri and German Rodrigo: arXiv: 2305.01686

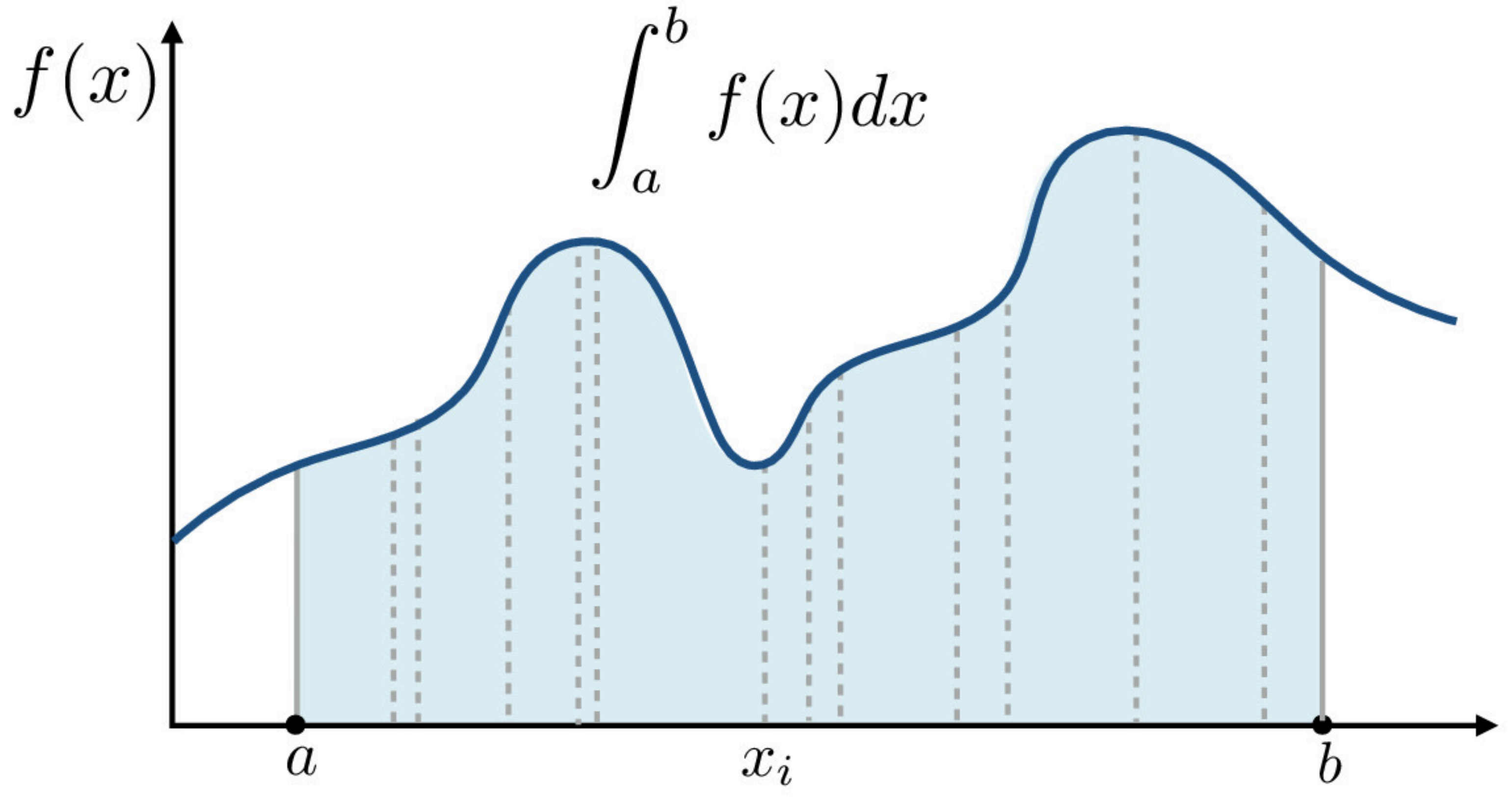
IFIC INSTITUT DE FÍSICA CORPUSCULAR

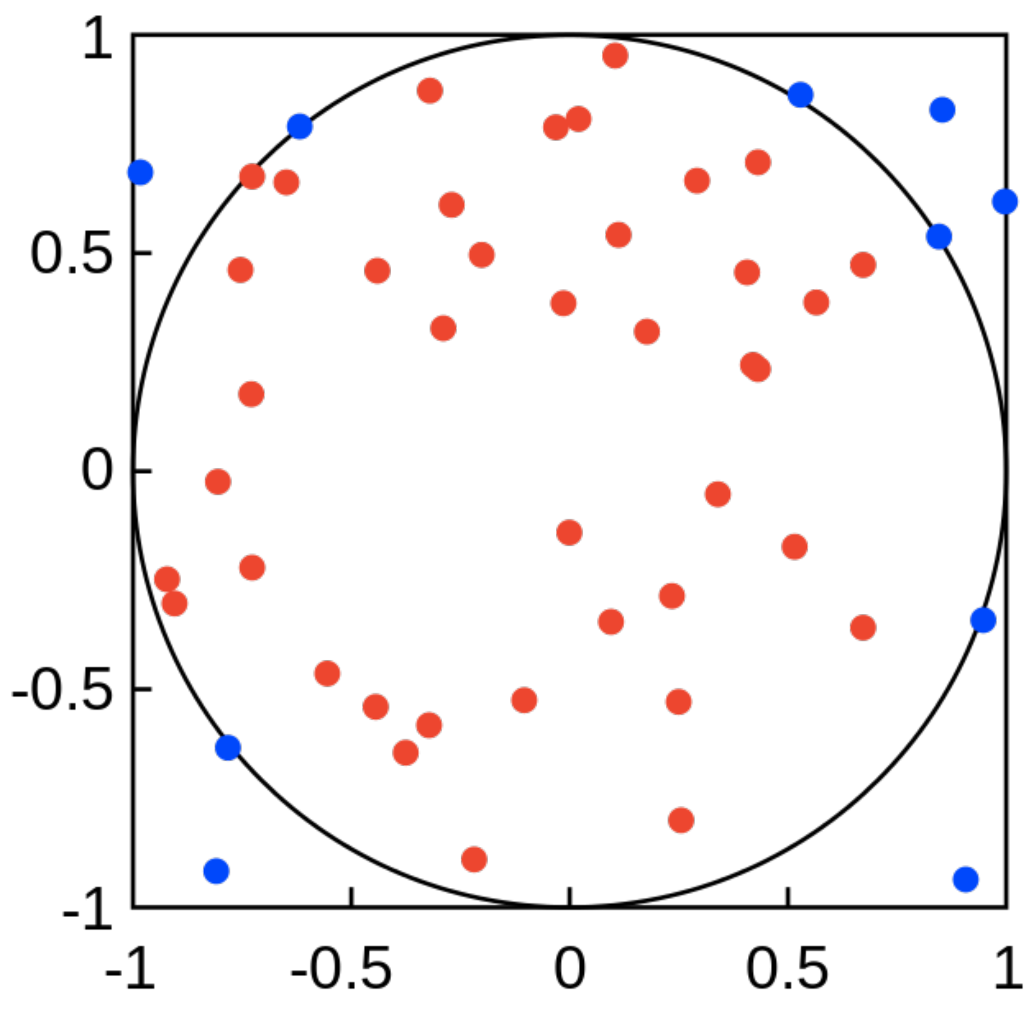
VNIVERSITAT DE VALÈNCIA

CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

CERN | QUANTUM TECHNOLOGY INITIATIVE

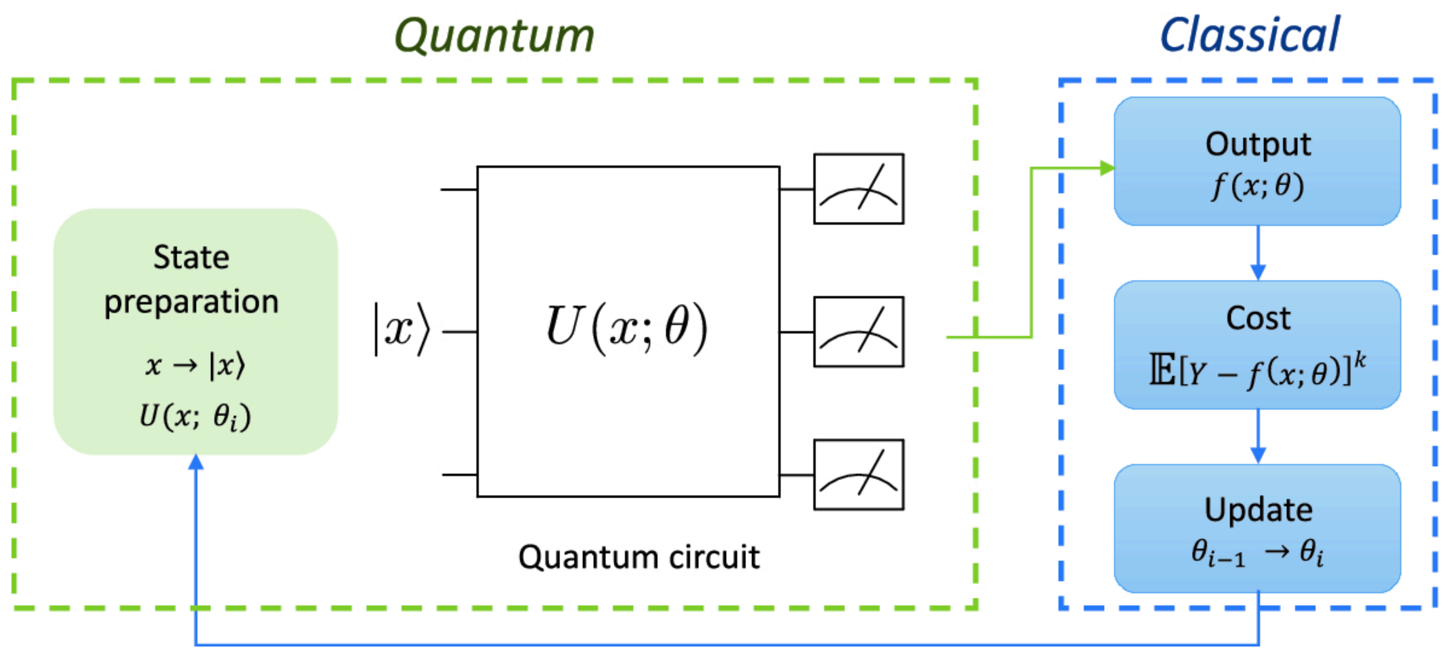
MONTE CARLO INTEGRATION

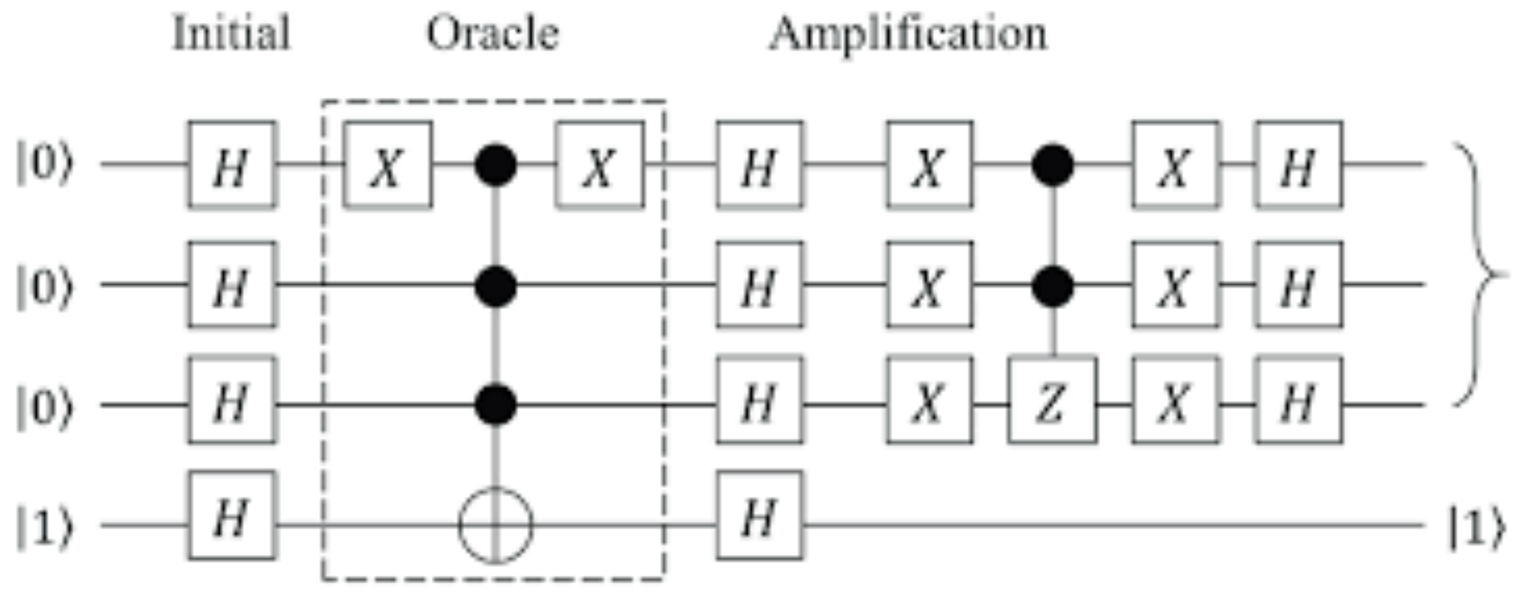




+

QUANTUM COMPUTING



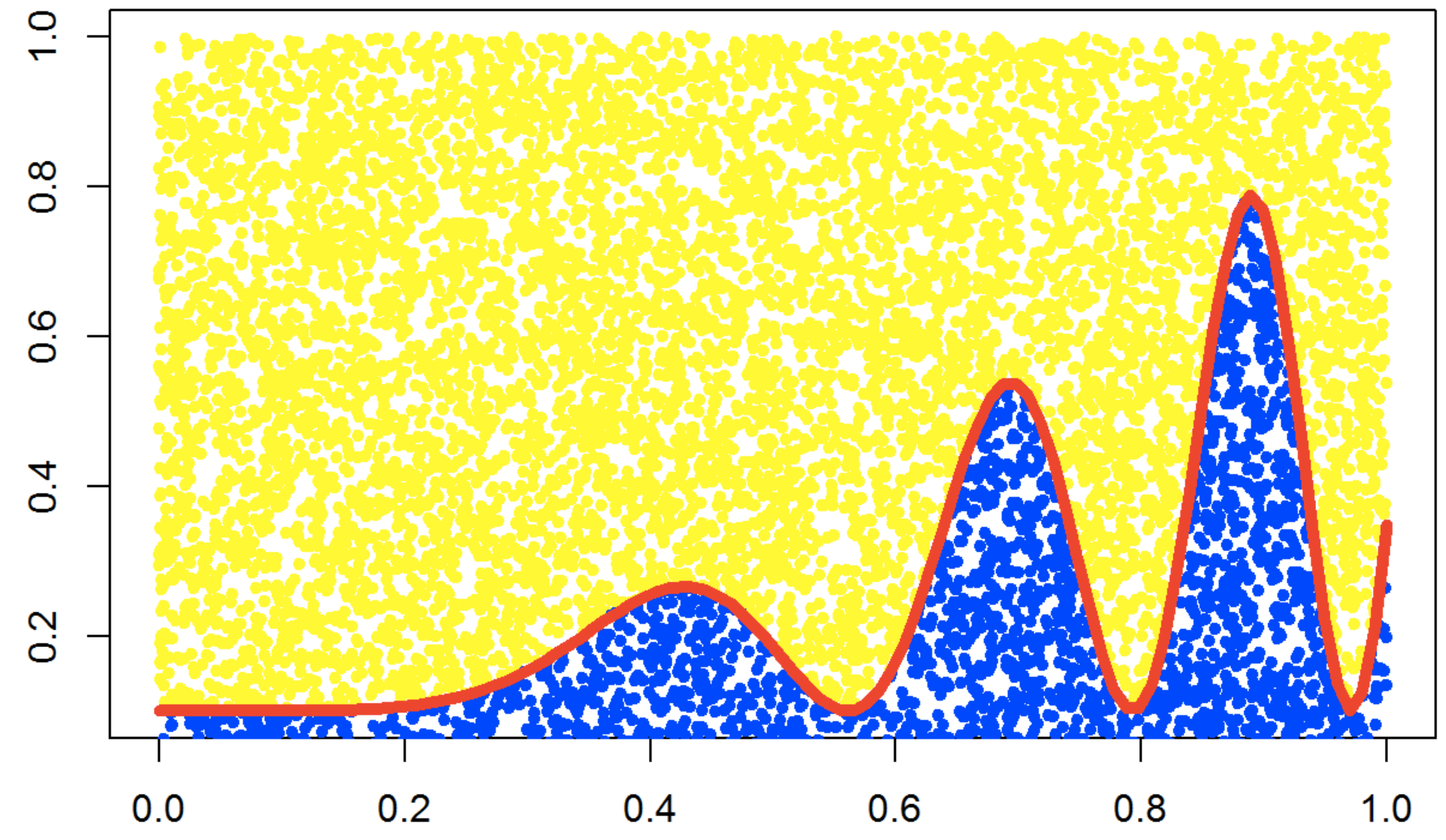


OUTLINE

- Motivation
- Quantum Amplitude Estimation
- Quantum Monte Carlo
- Fourier Quantum Monte Carlo Integration
- **Quantum Fourier Iterative Amplitude Estimation**
- Integration of a particle physics scattering process
- Conclusions and future work

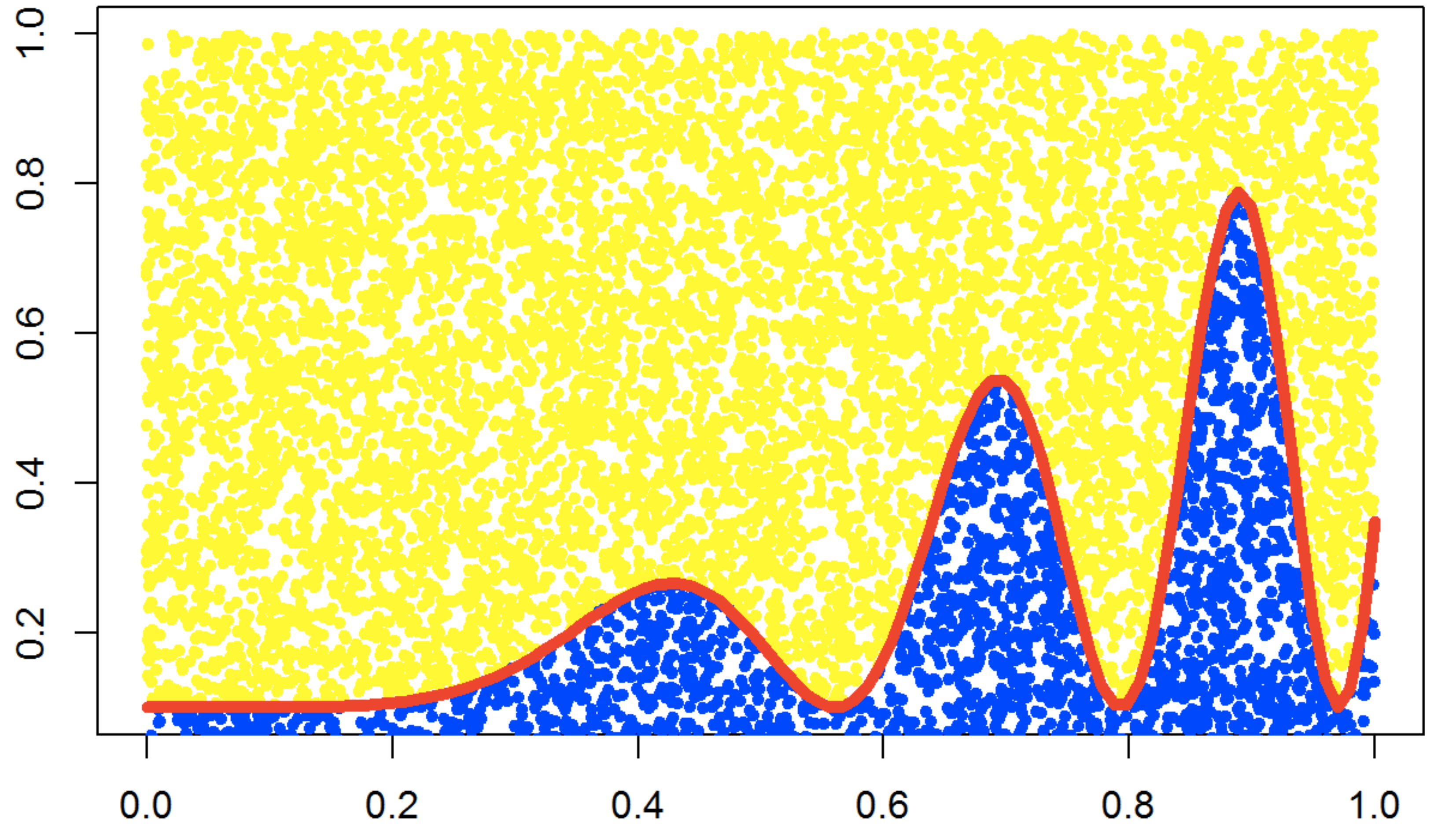
MOTIVATION

- *Monte Carlo* integration is a numerical method for approximating definite integrals using random sampling
- Widely used → particle physics, finance, AI
- Computationally demanding → some kind of integrals



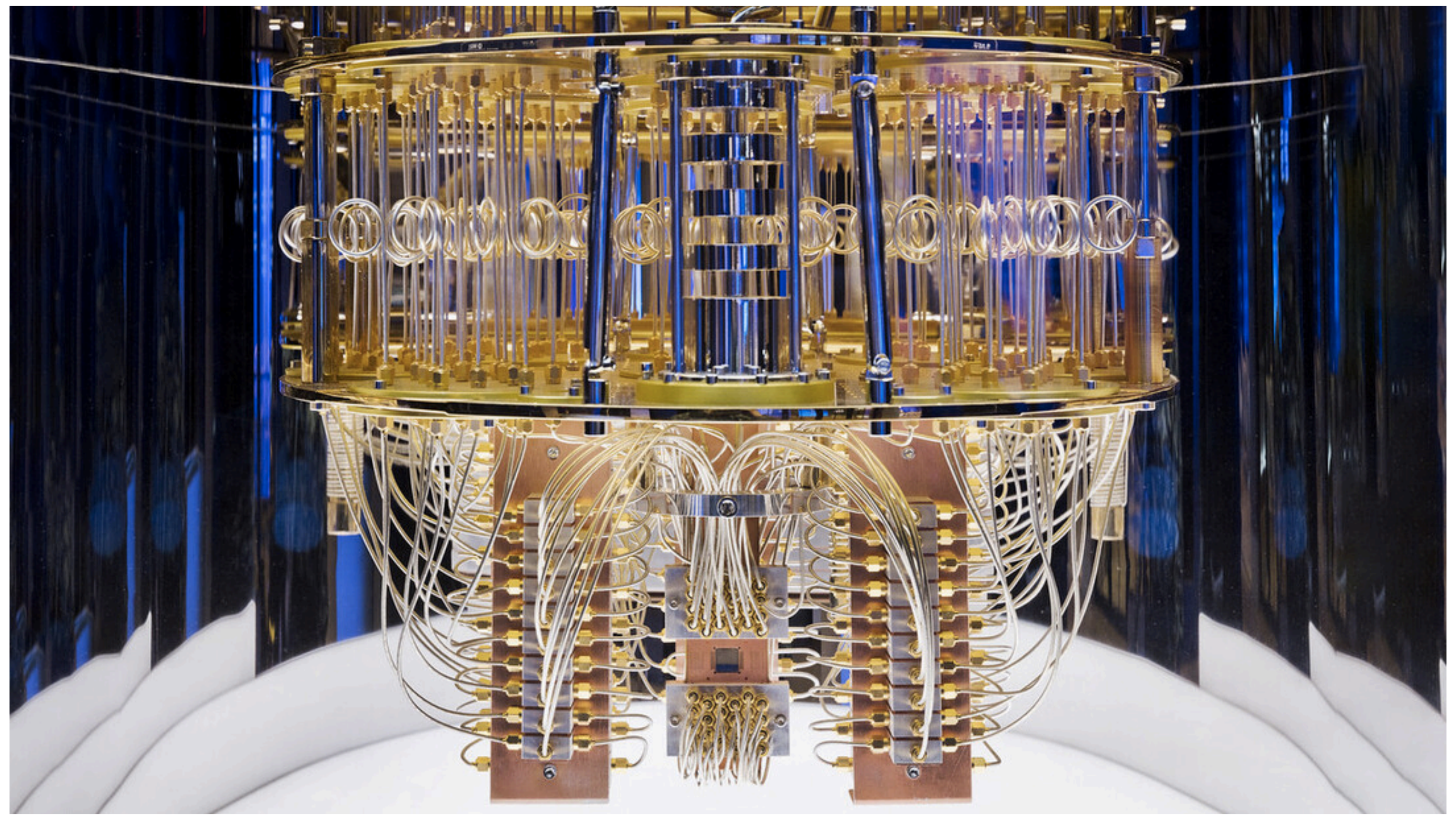
MOTIVATION

- *Monte Carlo* integration is a numerical method for approximating definite integrals using random sampling
- Widely used → particle physics, finance, AI
- Computationally demanding → some kind of integrals



Could Quantum Monte Carlo Integration do better?

- Quantum computers might improve the method
- *Quantum Monte Carlo Integration* exploits *Grover*'s algorithm: **quadratic speedup**



QUANTUM AMPLITUDE ESTIMATION

- Considering an operator $\mathcal{A}$: $\mathcal{A}|0\rangle = \sqrt{1-a}\,\underbrace{|\tilde{\psi}_0\rangle}_{\text{Bad state}} + \sqrt{a}\,\underbrace{|\tilde{\psi}_1\rangle}_{\text{Good state}}$
- Quantum Amplitude Estimation's (QAE) goal is to estimate the amplitude a of $|\tilde{\psi}_1\rangle$
- Using Grover's Amplitude Amplification: $\mathcal{Q} = -\mathcal{A}S_0\mathcal{A}^{-1}S_\chi,$

where $S_0|0\rangle_{n+1} = -|0\rangle_{n+1}$ and $S_\chi|1\rangle|\tilde{\psi}_1\rangle = -1\rangle|\tilde{\psi}_1\rangle$

QUANTUM AMPLITUDE ESTIMATION

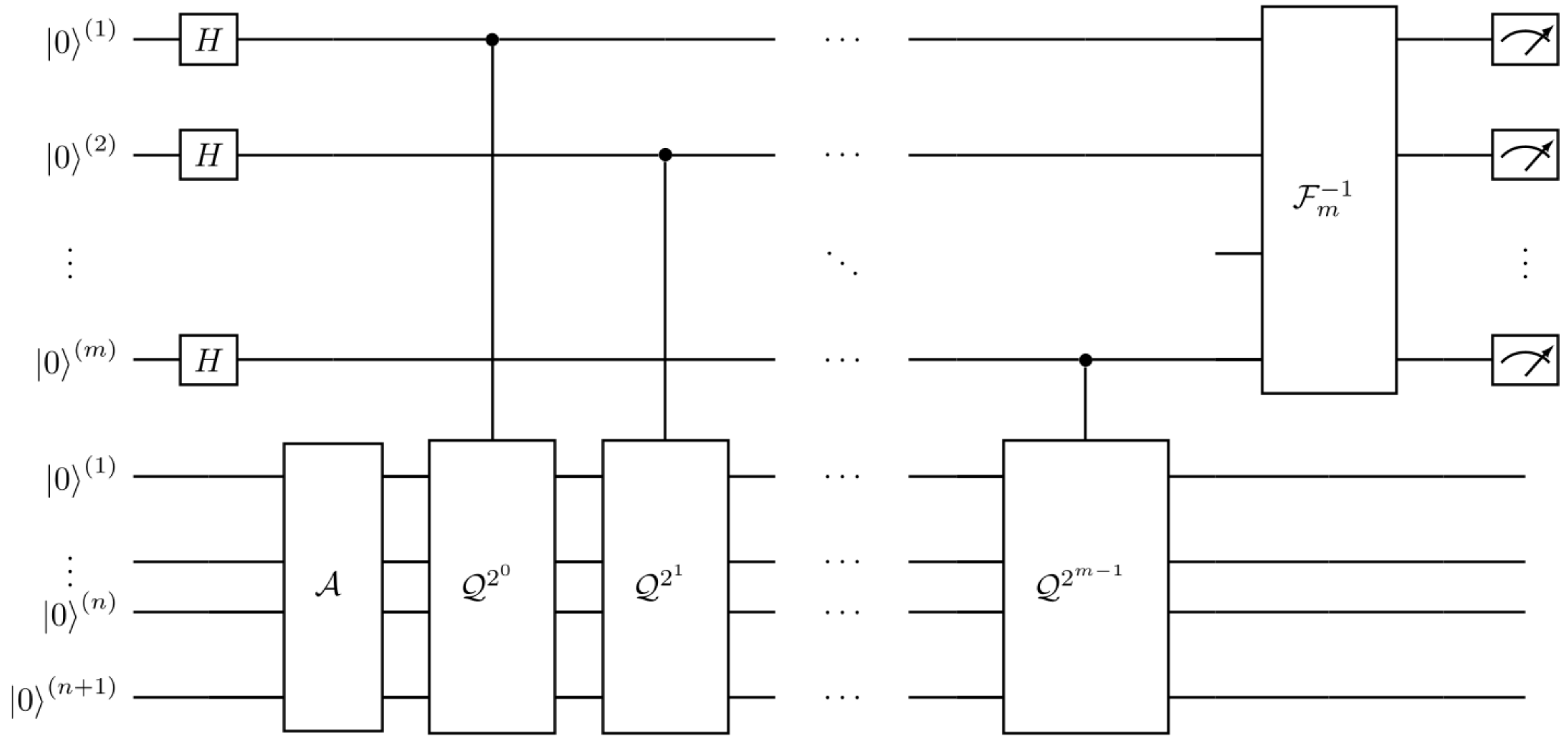
- Considering an operator $\mathscr{A}$: $\mathscr{A}\,|0\rangle = \sqrt{1-a}\,|\tilde{\psi}_0\rangle + \sqrt{a}\,|\tilde{\psi}_1\rangle$

  Bad state ($|\tilde{\psi}_0\rangle$) Good state ($|\tilde{\psi}_1\rangle$)

- Quantum Amplitude Estimation's (QAE) goal is to estimate the amplitude a of $|\tilde{\psi}_1\rangle$
- Using Grover's Amplitude Amplification: $\mathscr{Q} = -\mathscr{A}S_0\mathscr{A}^{-1}S_\chi$,

  where $S_0|0\rangle_{n+1} = -|0\rangle_{n+1}$ and $S_\chi|1\rangle|\tilde{\psi}_1\rangle = -1\rangle|\tilde{\psi}_1\rangle$

- Original version:



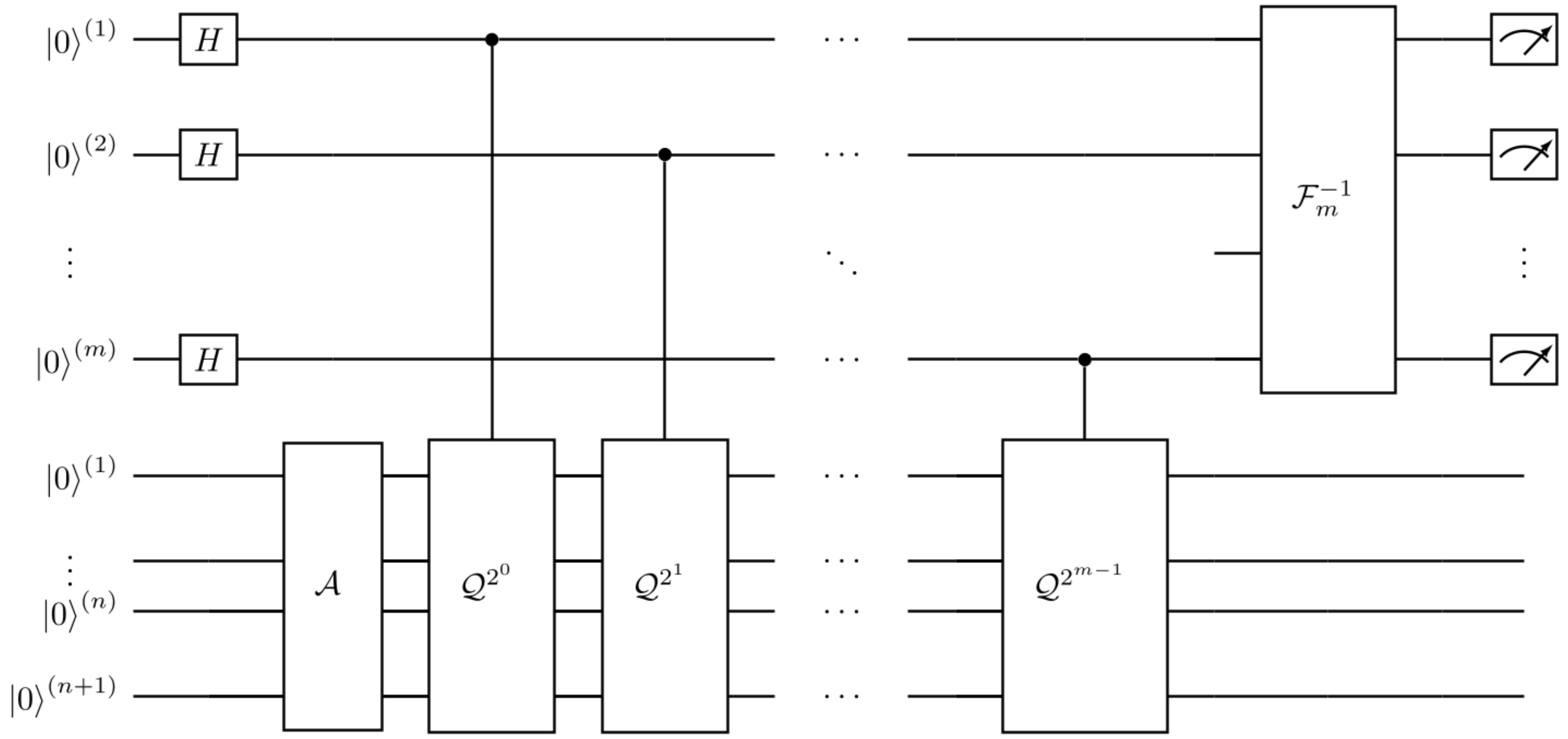
Brassard, Hoyer, Mosca, Tapp, [0005055]

QUANTUM AMPLITUDE ESTIMATION

- Considering an operator $\mathscr{A}$: $\mathscr{A}|0\rangle = \sqrt{1-a}\,|\tilde{\psi}_0\rangle + \sqrt{a}\,|\tilde{\psi}_1\rangle$

  Bad state ($|\tilde{\psi}_0\rangle$) Good state ($|\tilde{\psi}_1\rangle$)

- Quantum Amplitude Estimation's (QAE) goal is to estimate the amplitude a of $|\tilde{\psi}_1\rangle$
- Using Grover's Amplitude Amplification: $\mathscr{Q} = -\mathscr{A}S_0\mathscr{A}^{-1}S_\chi$,
- Original version:



Brassard, Hoyer, Mosca, Tapp, 0005055

where $S_0|0\rangle_{n+1} = -|0\rangle_{n+1}$ and $S_\chi|1\rangle|\tilde{\psi}_1\rangle = -1\rangle|\tilde{\psi}_1\rangle$

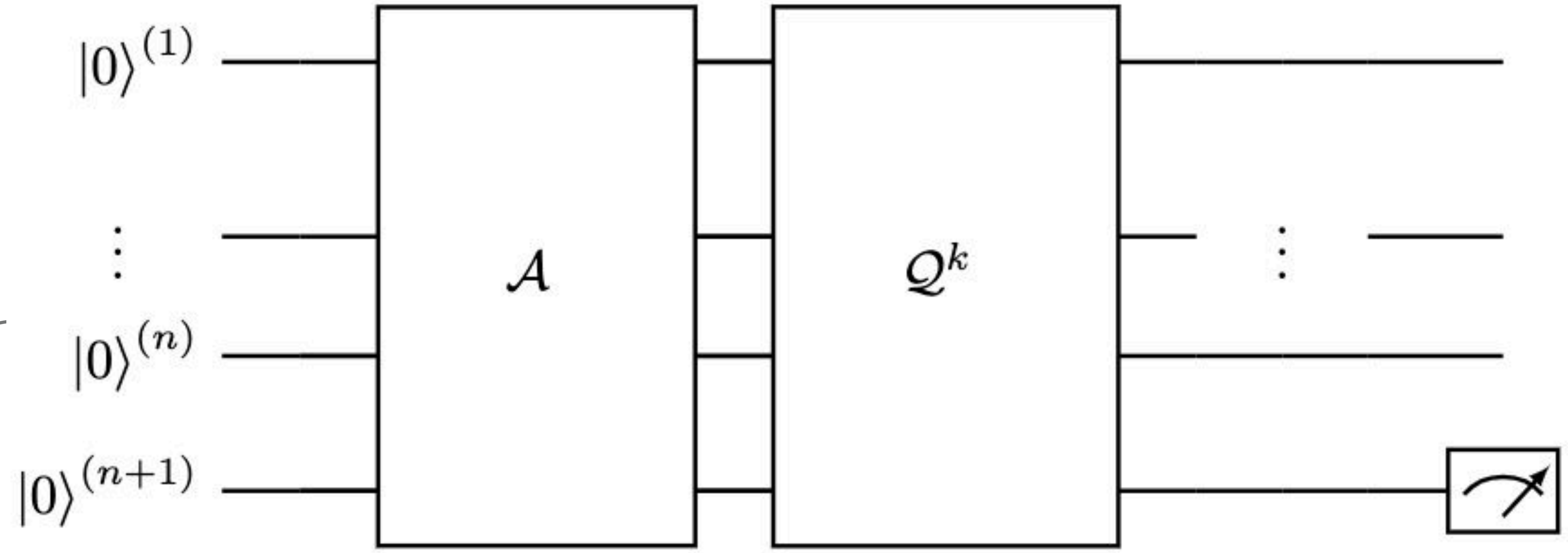
Drawback:

Requires inverse Quantum Fourier transform (QFT)

Computationally expensive

QUANTUM AMPLITUDE ESTIMATION

➤ To avoid QFT different versions of QAE are proposed:



Maximum Likelihood Amplitude Estimation:

Suzuki, et al. QIP 19, 75 (2020)

Variational quantum amplitude estimation:

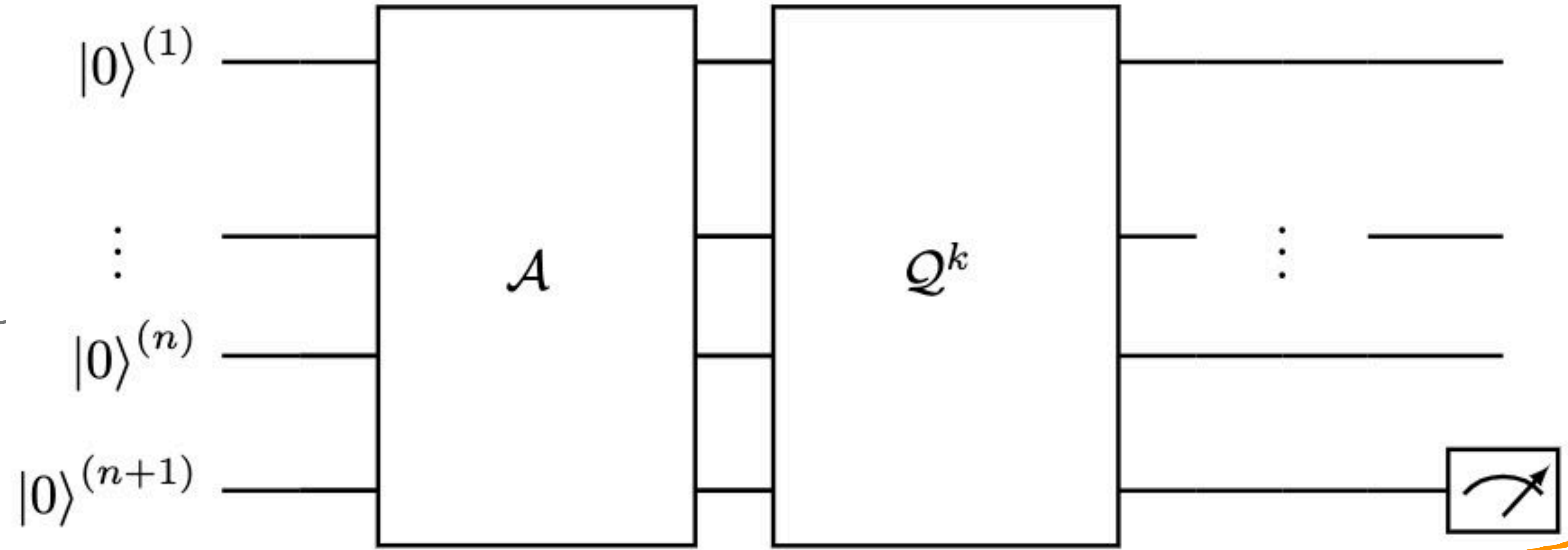
Plekhanov, Rosenkranz, Fiorentini, Lubasch Q6, 670 (2022)

Iterative Quantum Amplitude Estimation:

Grinko, Gacon, Zoufal, Woerner npj QI 7, 52 (2021)

QUANTUM AMPLITUDE ESTIMATION

- To avoid QFT different versions of QAE are proposed:



Maximum Likelihood Amplitude Estimation:

Suzuki, et al. QIP 19, 75 (2020)

Variational quantum amplitude estimation:

Plekhanov, Rosenkranz, Fiorentini, Lubasch Q6, 670 (2022)

Iterative Quantum Amplitude Estimation:

Grinko, Gacon, Zoufal, Woerner npj QI 7, 52 (2021)

Demonstrated speed up!

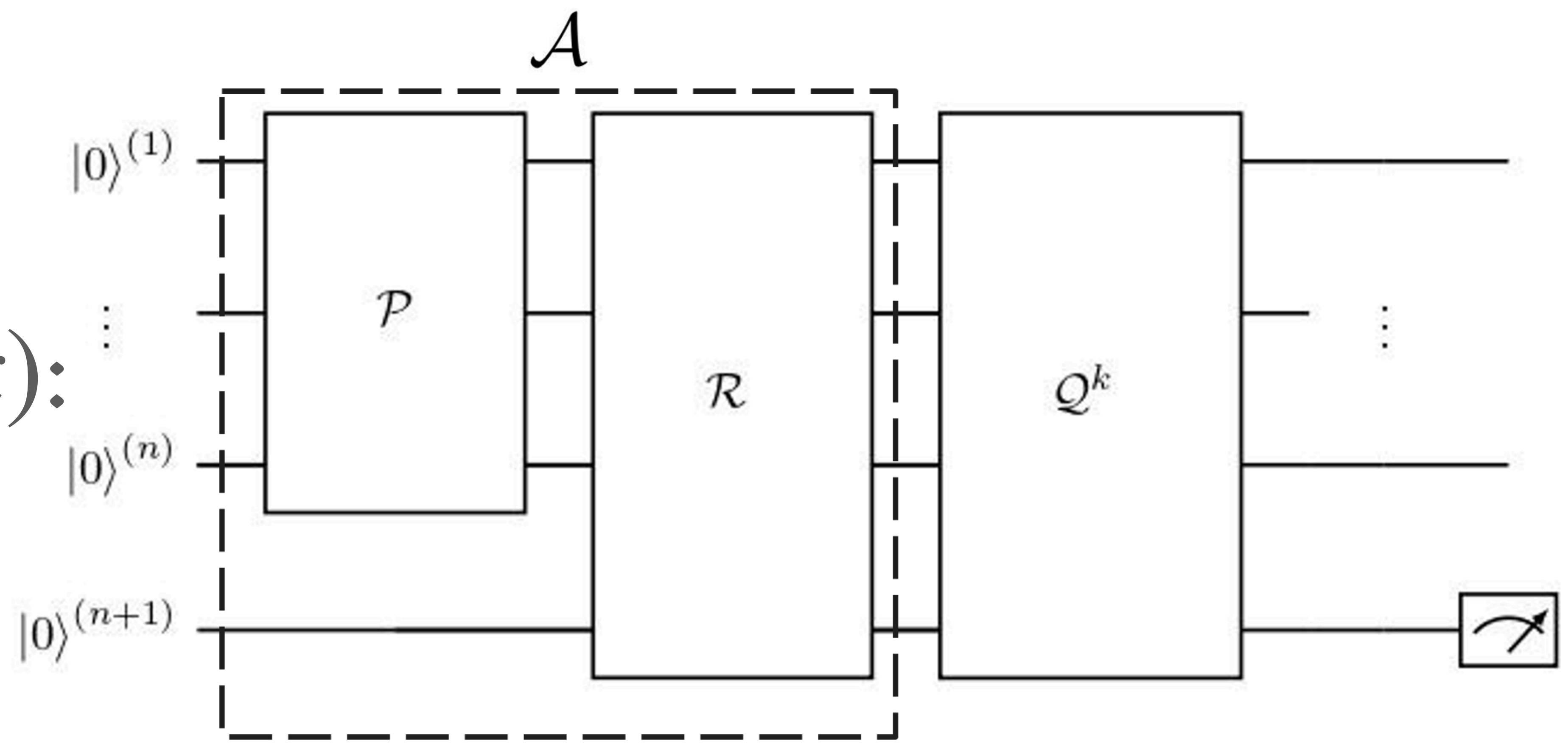
QUANTUM MONTE CARLO INTEGRATION

- Quantum Monte Carlo's goal is computing the expected value of $0 \leq f(x) \leq 1$ defined for n-bit input $x \in \{0,1\}^n$ with probability distribution $p(x)$:

$$\int p(x)f(x)dx \quad \longrightarrow \quad \mathbb{E}[f(x)] = \sum_{x=0}^{2^n-1} p(x)f(x)$$

QUANTUM MONTE CARLO INTEGRATION

- Quantum Monte Carlo's goal is computing the expected value of $0 \leq f(x) \leq 1$ defined for n-bit input $x \in \{0,1\}^n$ with probability distribution $p(x)$:



$$\int p(x)f(x)dx \quad \longrightarrow \quad \mathbb{E}[f(x)] = \sum_{x=0}^{2^n-1} p(x)f(x)$$

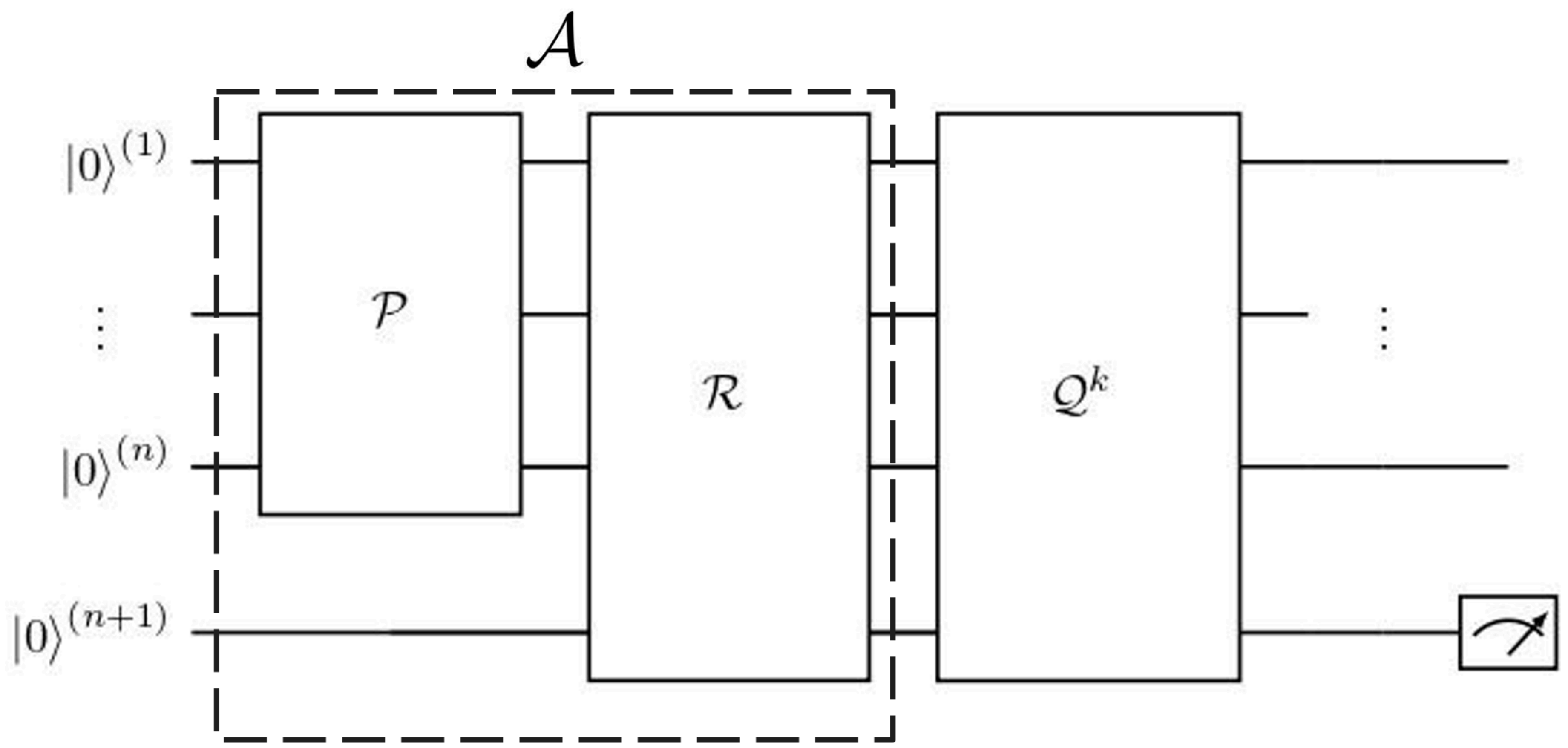
- Smartly define $\mathscr{A} = \mathscr{R}(\mathscr{P} \otimes \mathrm{I}^1)$, where $\mathscr{P}$ and $\mathscr{R}$:

$$\mathscr{P}|0\rangle_n = \sum_{x=0}^{2^n-1} \sqrt{p(x)}\,|x\rangle_n, \qquad \mathscr{R}|x\rangle_n|0\rangle = |x\rangle_n\left(\sqrt{f(x)}\,|1\rangle + \sqrt{1-f(x)}\,|0\rangle\right)$$

- The state will be: $|\psi\rangle = \mathscr{A}|0\rangle_{n+1} = \sqrt{a}\,\underbrace{|\tilde{\psi}_1\rangle}_{\text{Good state}}|1\rangle + \sqrt{1-a}\,\underbrace{|\tilde{\psi}_0\rangle}_{\text{Bad state}}|0\rangle$ where $a = \mathbb{E}[f(x)]$

FOURIER QUANTUM MONTE CARLO INTEGRATION

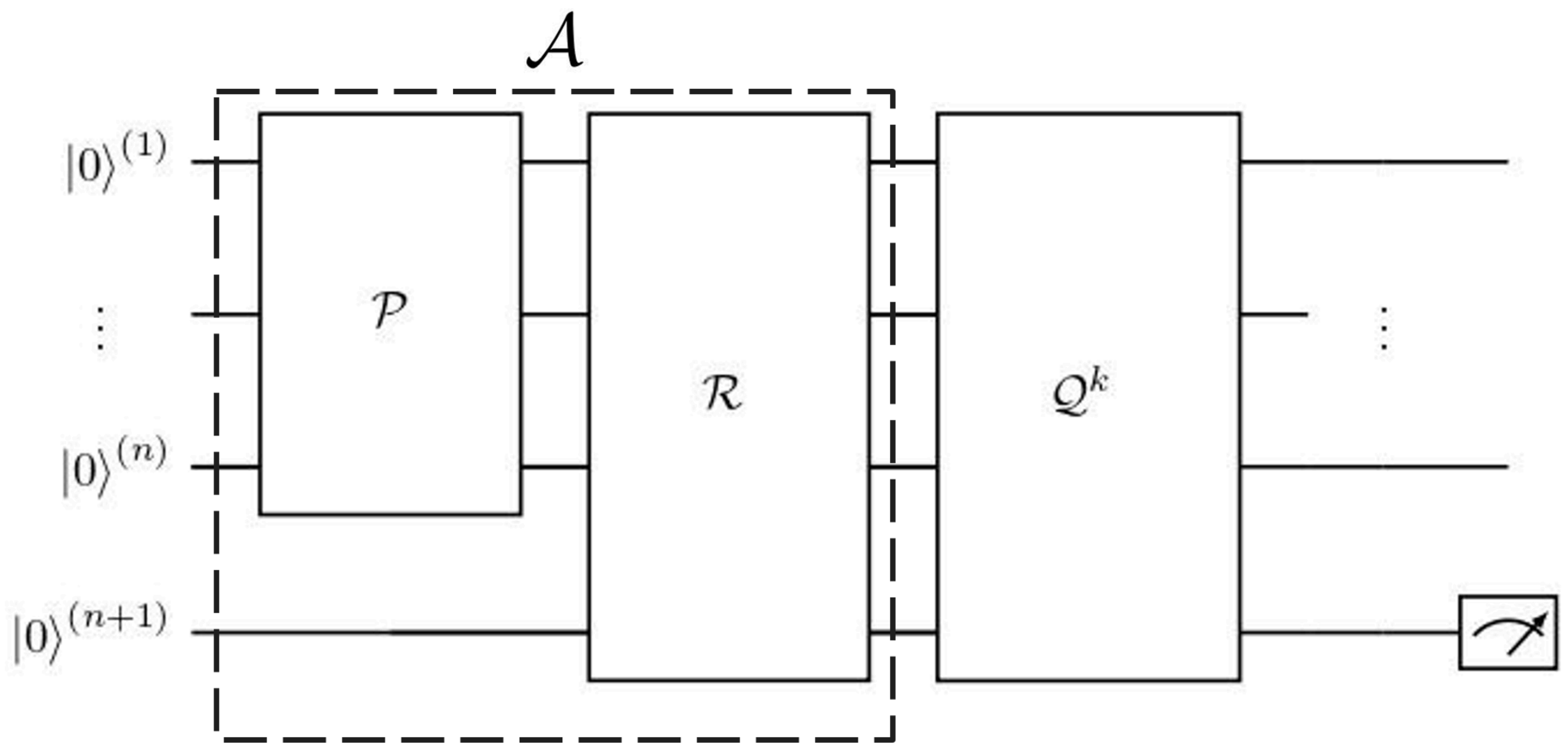
- **Problem**: encoding $f(x)$ into $\mathscr{R}$ quantum circuit
  - Lots of quantum operations $\longrightarrow$ unfeasible



FOURIER QUANTUM MONTE CARLO INTEGRATION

- **Problem**: encoding $f(x)$ into $\mathscr{R}$ quantum circuit
  - Lots of quantum operations $\longrightarrow$ unfeasible
- **Solution:** decomposing $f(x)$ into **Fourier series** since encoding $\sin(ax + b)^2$ is simpler:

$$f(x) = c + \sum_{n=1}^{\infty} (a_n \cos(n\omega x) + b_n \sin(n\omega x))$$

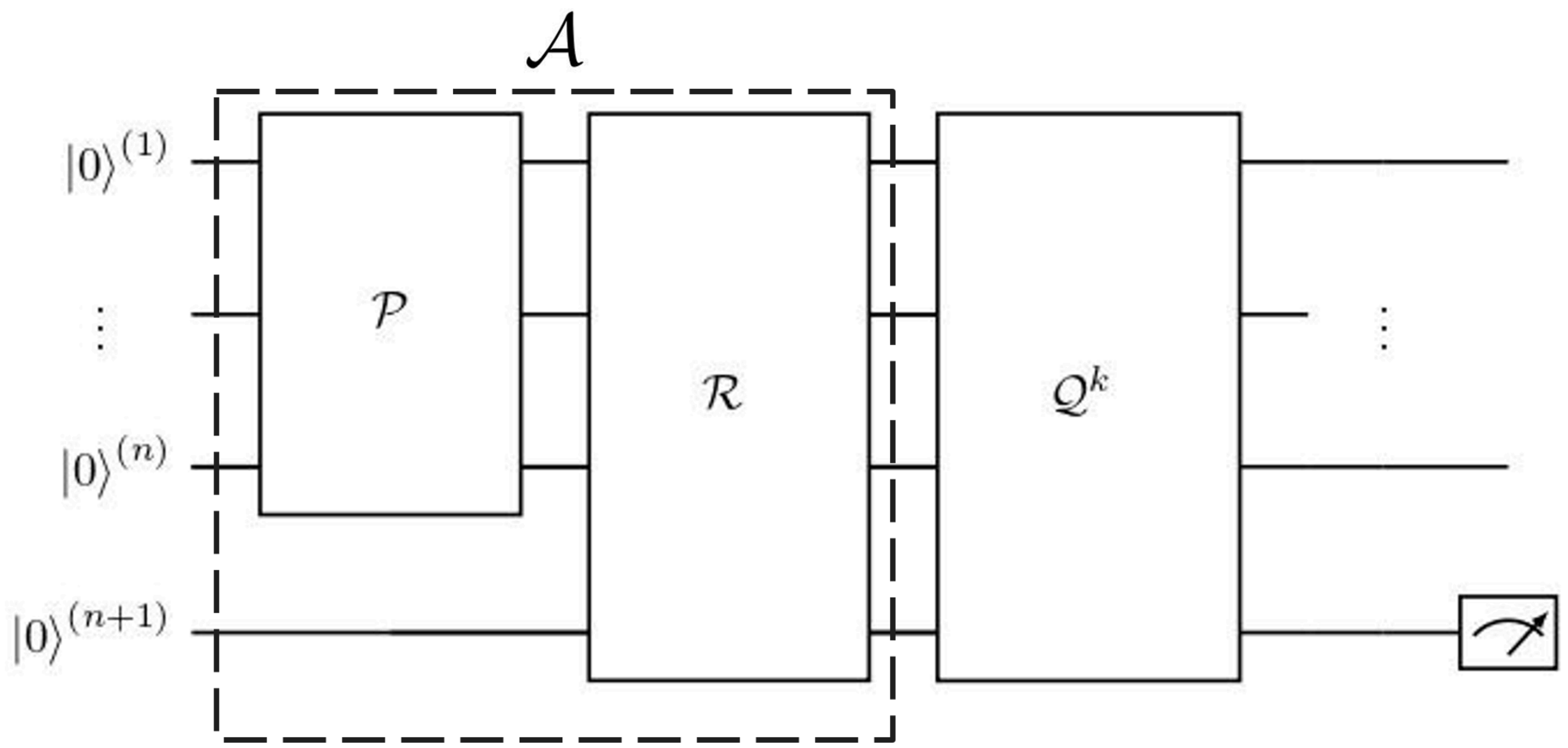


FQMCI: Herbert Q6, 823 (2022)

FOURIER QUANTUM MONTE CARLO INTEGRATION

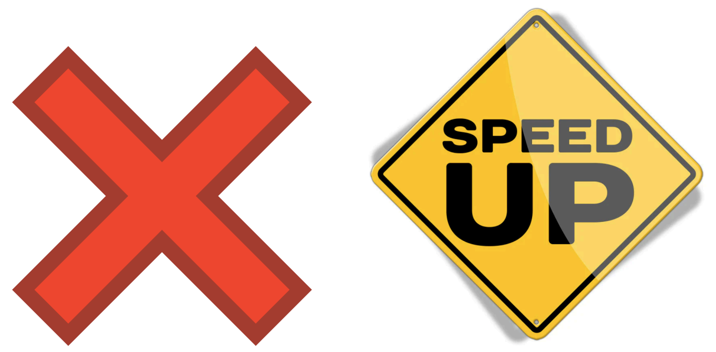
- **Problem**: encoding $f(x)$ into $\mathscr{R}$ quantum circuit
  - Lots of quantum operations $\longrightarrow$ unfeasible
- **Solution:** decomposing $f(x)$ into **Fourier series** since encoding $\sin(ax+b)^2$ is simpler:

$$f(x) = c + \sum_{n=1}^{\infty} (a_n \cos(n\omega x) + b_n \sin(n\omega x))$$



FQMCI: Herbert Q6, 823 (2022)

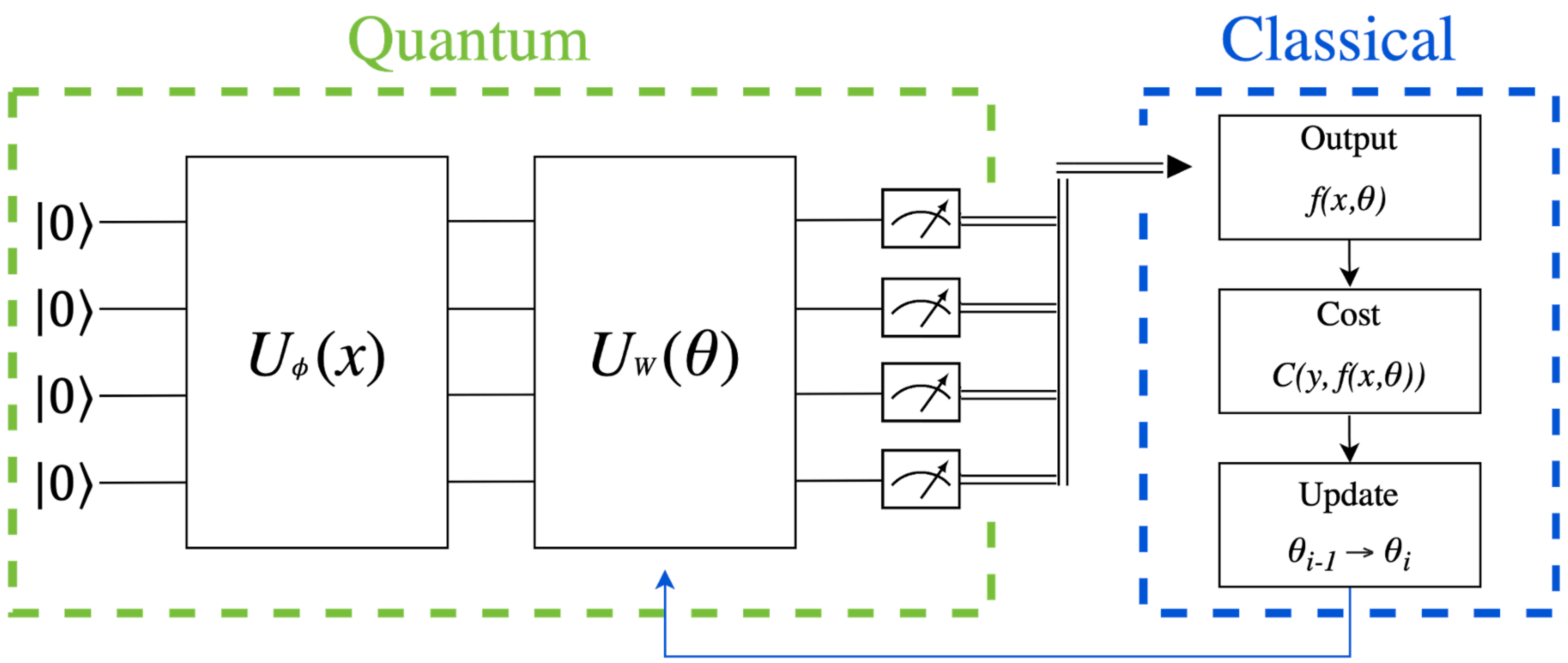
- FQMCI involves **other issue** $\longrightarrow$ How calculate $\{a_n, b_n\}$?
  - Not addresses this matter $\longrightarrow$ assumes precomputed or analytically computable $\longrightarrow$ 

  - In general $\longrightarrow$ needs numerical integration $\longrightarrow$ **shift in computational load** $\longrightarrow$ 

QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

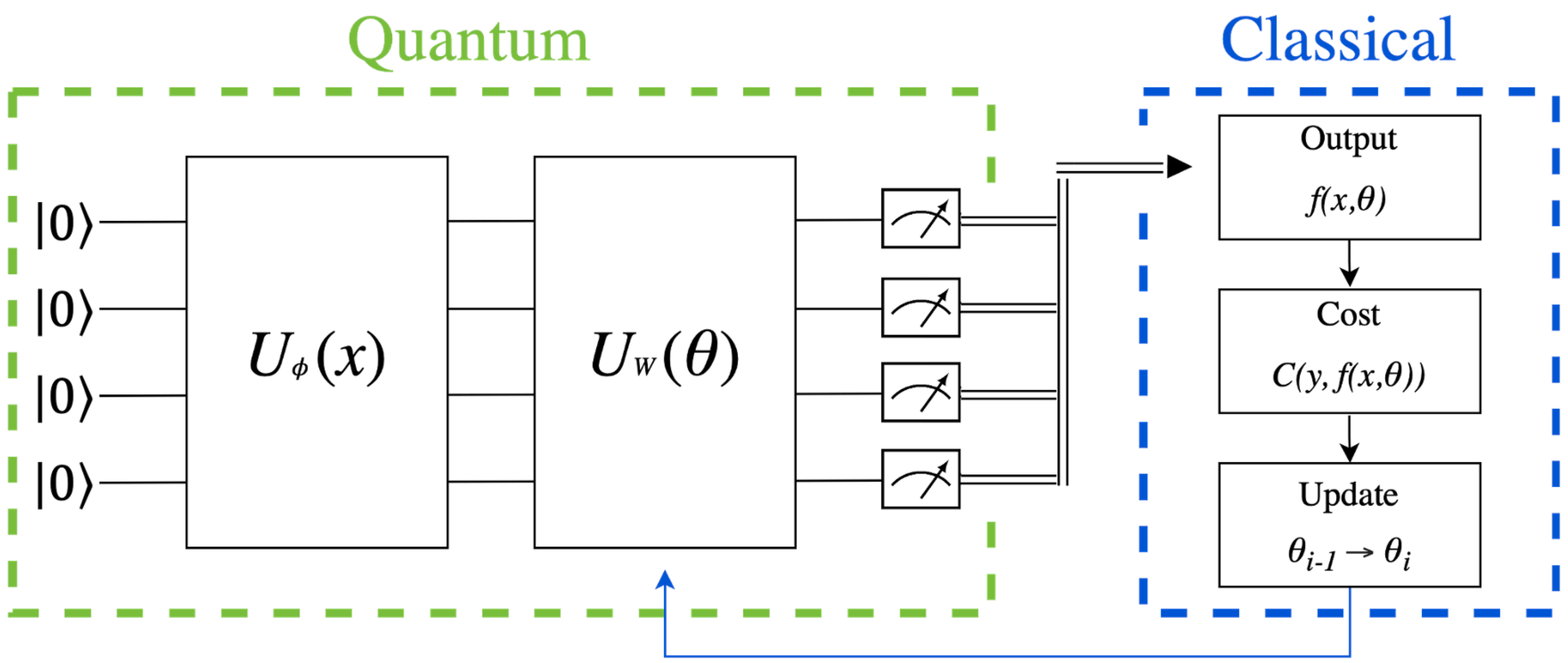
- **Main idea**: based on FQMCI but uses a Variational Quantum Circuit (VQC) to obtain Fourier series



QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- **Main idea**: based on FQMCI but uses a Variational Quantum Circuit (VQC) to obtain Fourier series



- VQC ⟶ Quantum Neural Network (QNN)
- Expectation value corresponds to a truncated Fourier series:

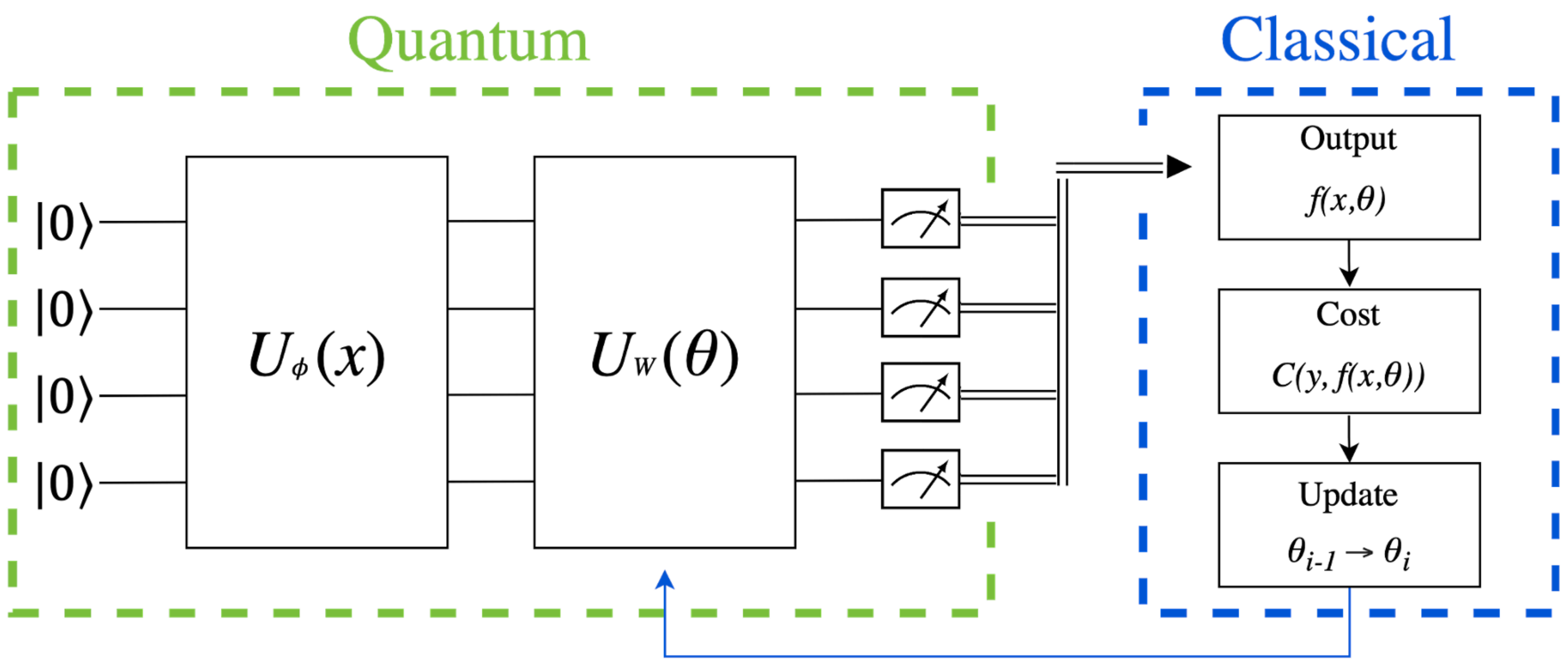
$$\langle M((\vec{x}, \vec{\theta})\rangle = \sum_{\vec{\omega} \in \Omega} c(\vec{\theta})_{\vec{\omega}} e^{i\vec{x}\cdot\vec{\omega}}$$

Schuld, Sweke, Meyer PRA 103, 032430 (2021)

QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- **Main idea**: based on FQMCI but uses a Variational Quantum Circuit (VQC) to obtain Fourier series



- VQC ⟶ Quantum Neural Network (QNN)
- Expectation value corresponds to a truncated Fourier series:

$$\langle M((\vec{x}, \vec{\theta})\rangle = \sum_{\vec{\omega}\in\Omega} c(\vec{\theta})_{\vec{\omega}} e^{i\vec{x}\cdot\vec{\omega}}$$

Schuld, Sweke, Meyer PRA 103, 032430 (2021)

- After fitting $f(\vec{x})$ with QNN ⟶ obtain Fourier coeffs $c(\vec{\theta}_{opt})_{\vec{\omega}}$ ⟶ IQAE to **every** trigonometric piece

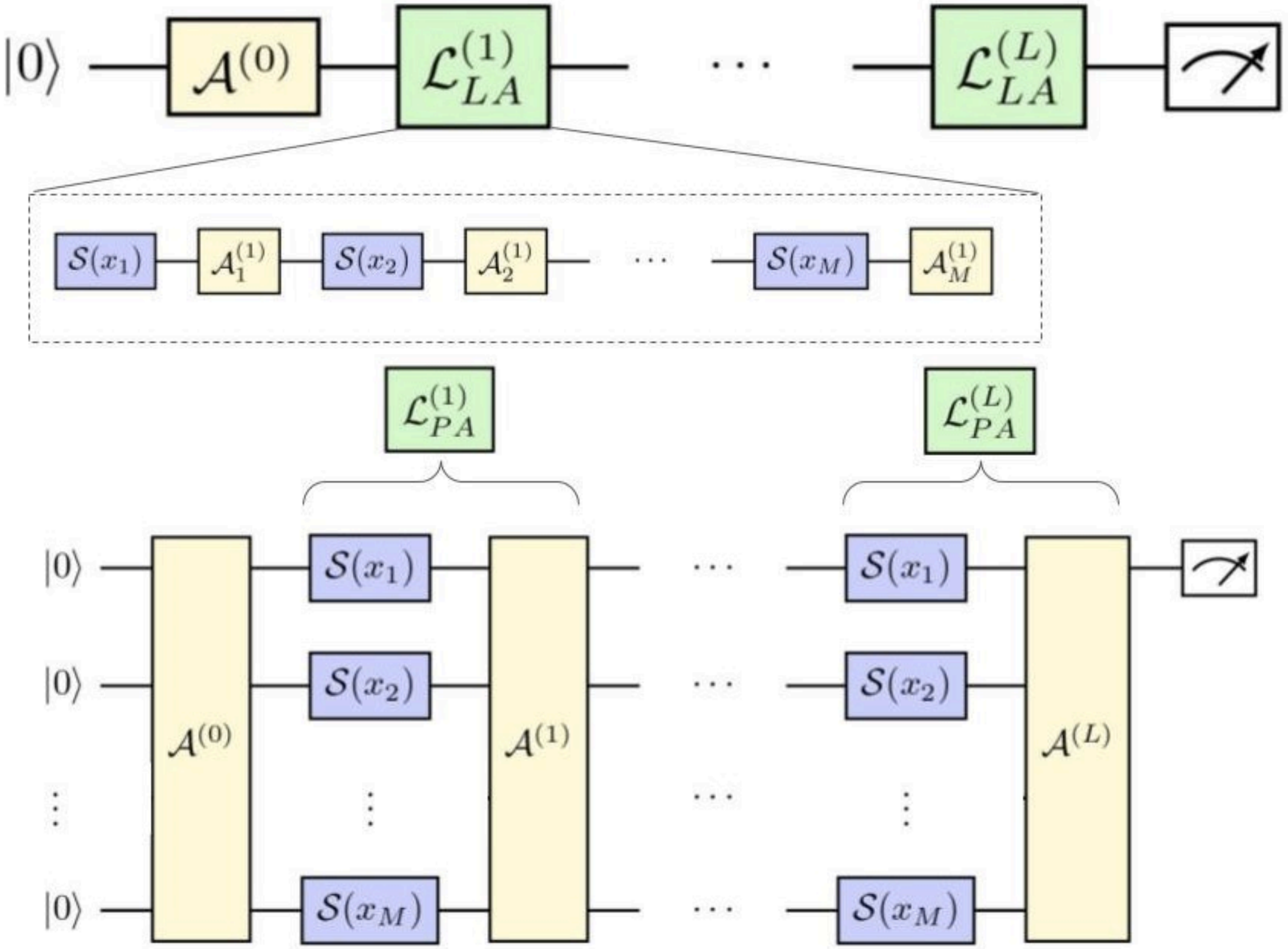
QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- QNN structure: $\left\{ \begin{array}{l} \text{Encoding gates: } S(x_i) \\ \text{Trainable gates: } A(\vec{\theta}_l) \end{array} \right\}$ Mixed layers $L^{(l)}$

Data re-uploading JPérez-Salinas,Cervera-Lierta, Gil-Fuster, Latorre Q4, 226 (2020)

- Linear Ansatz:



$\longrightarrow$ 1 qubit

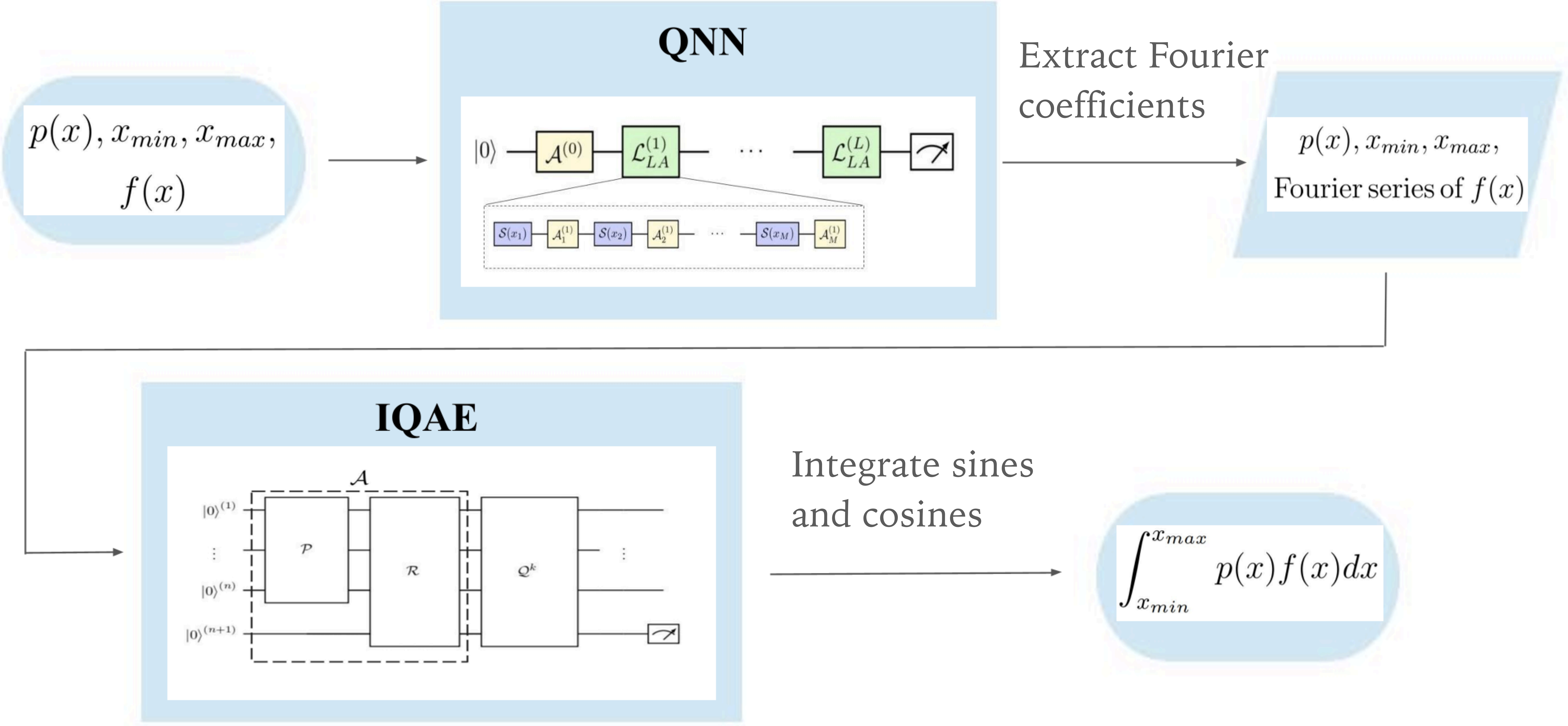
- Parallel Ansatz:

$\longrightarrow$ M qubits

QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION

QFIAE JML, Grossi, Cieri, Rodrigo, [2305.01686]

- Workflow of QFIAE for 1-D functions:



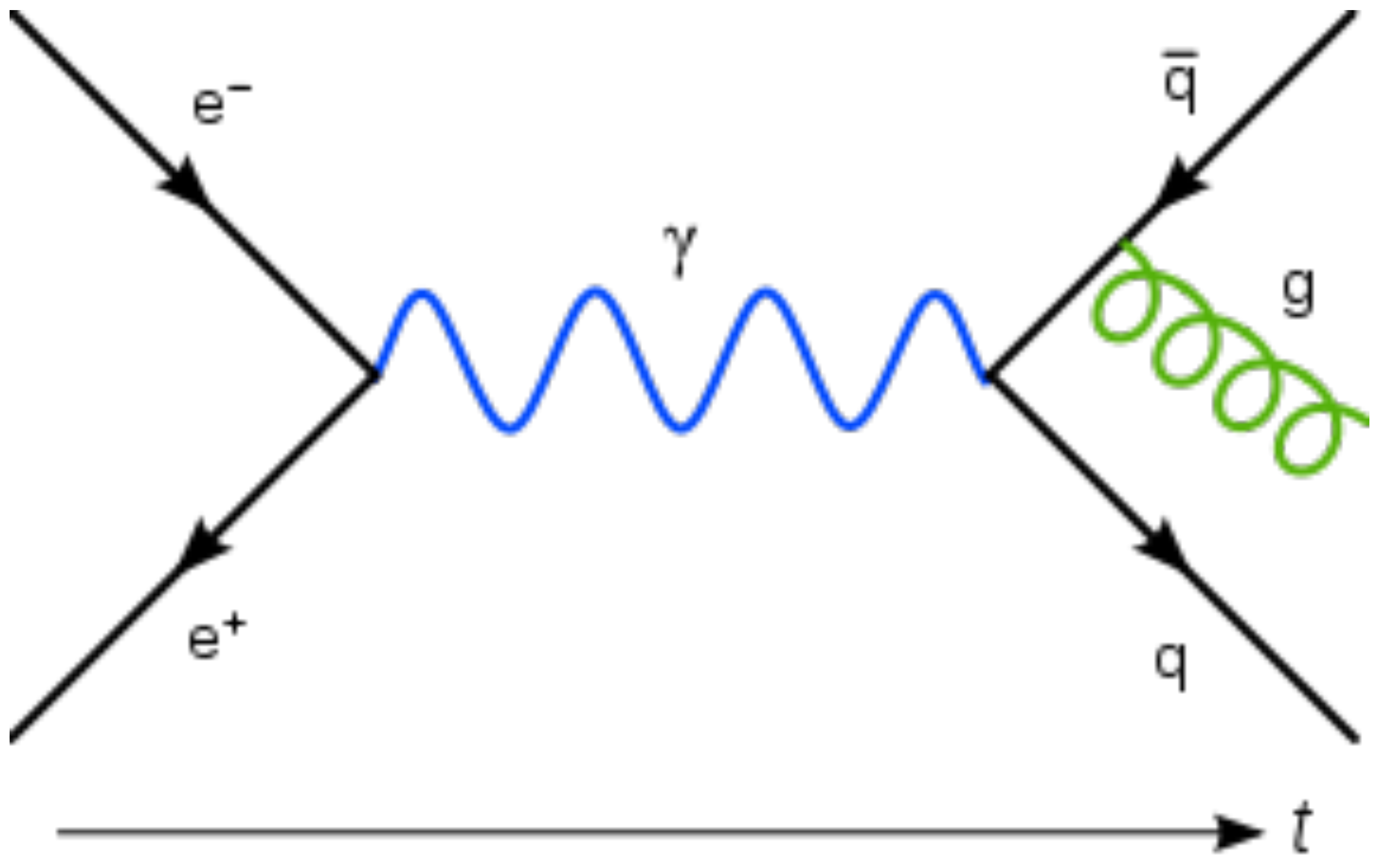
INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- Scattering process: $e^- e^+ \to q\bar{q}$ in QED

Whose cross section is:

$$\sigma \sim \int_{-1}^{1} \int_{0}^{2\pi} d\cos\theta d\phi (1 + \cos^2\theta)$$



INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- Scattering process: $e^-e^+ \rightarrow q\bar{q}$ in QED

Whose cross section is:

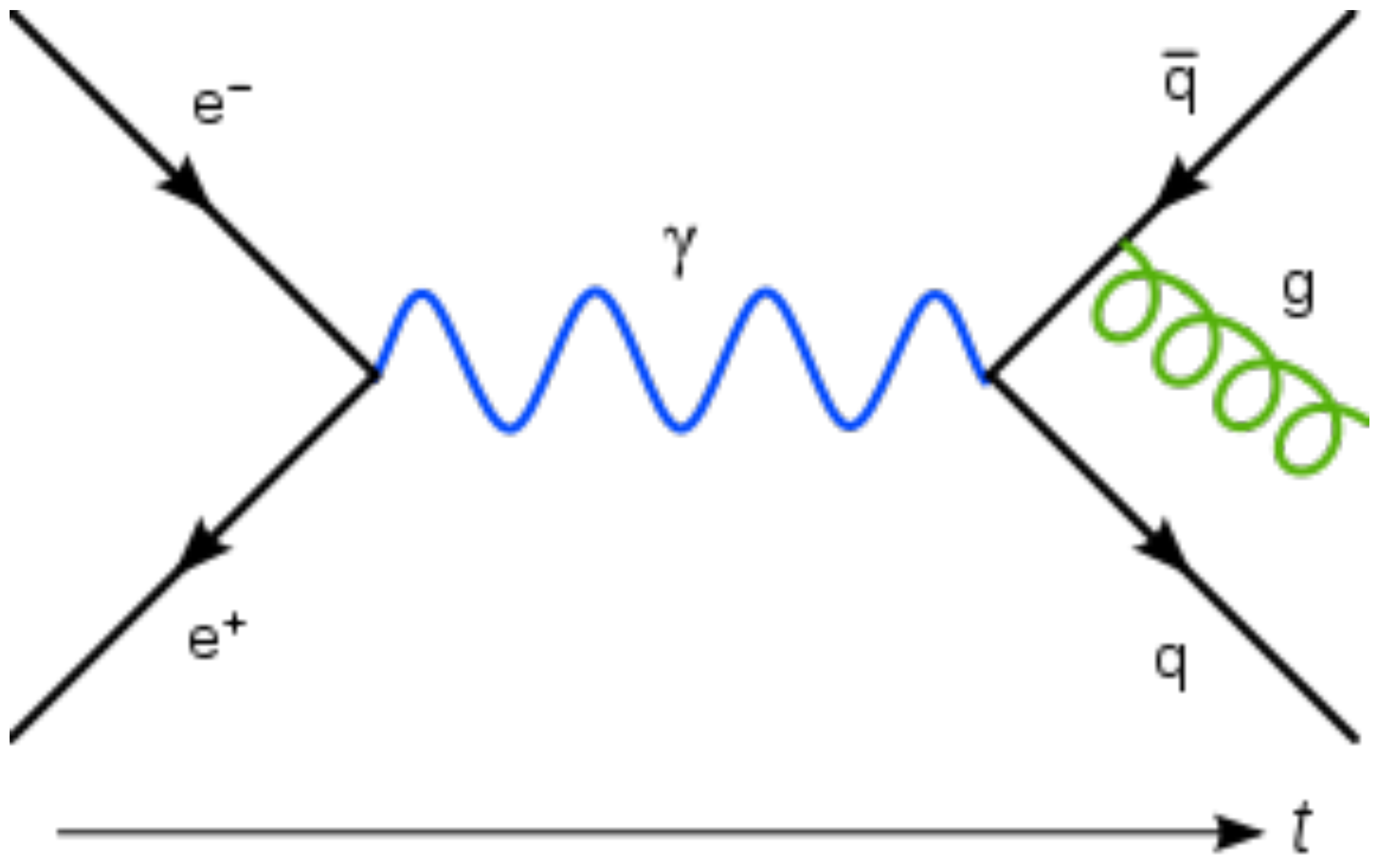
$$\sigma \sim \int_{-1}^{1}\int_{0}^{2\pi} d\cos\theta d\phi(1+\cos^2\theta)$$

$x = \cos\theta$

$p(x) = 1/2^n \qquad f(x) = 1 + x^2$

$x_{min} = 0 \quad x_{max} = 1$

QFIAE



INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- Scattering process: $e^-e^+ \to q\bar{q}$ in QED

Whose cross section is:

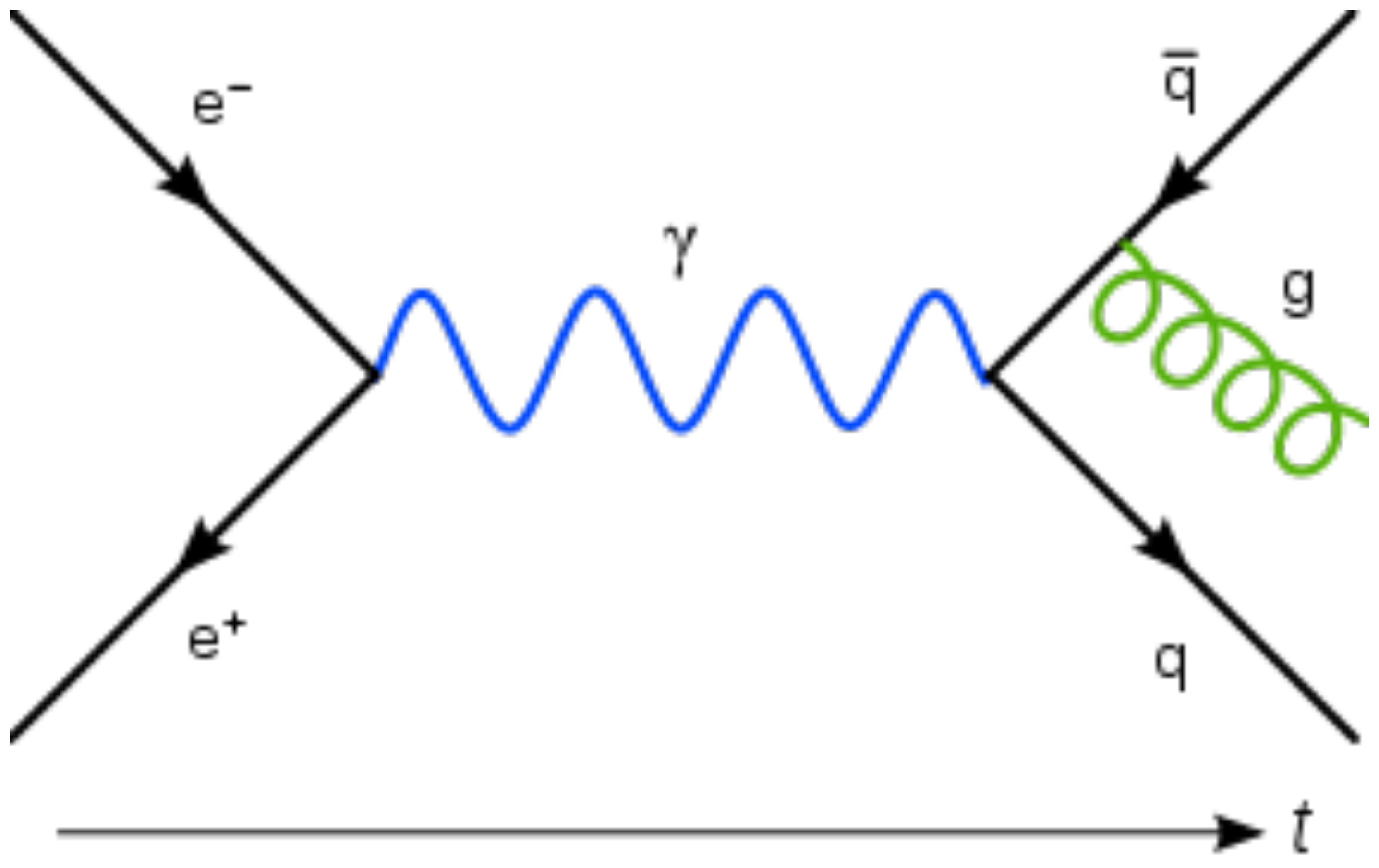
$$\sigma \sim \int_{-1}^{1} \int_{0}^{2\pi} d\cos\theta d\phi (1+\cos^2\theta)$$

$x = \cos\theta$

$p(x) = 1/2^n \qquad f(x) = 1 + x^2$

$x_{min} = 0 \quad x_{max} = 1$

QFIAE



Agliardi, Grossi, Pellen, Prati, PLB 832, 137228 (2022)

Using QGAN+IQAE

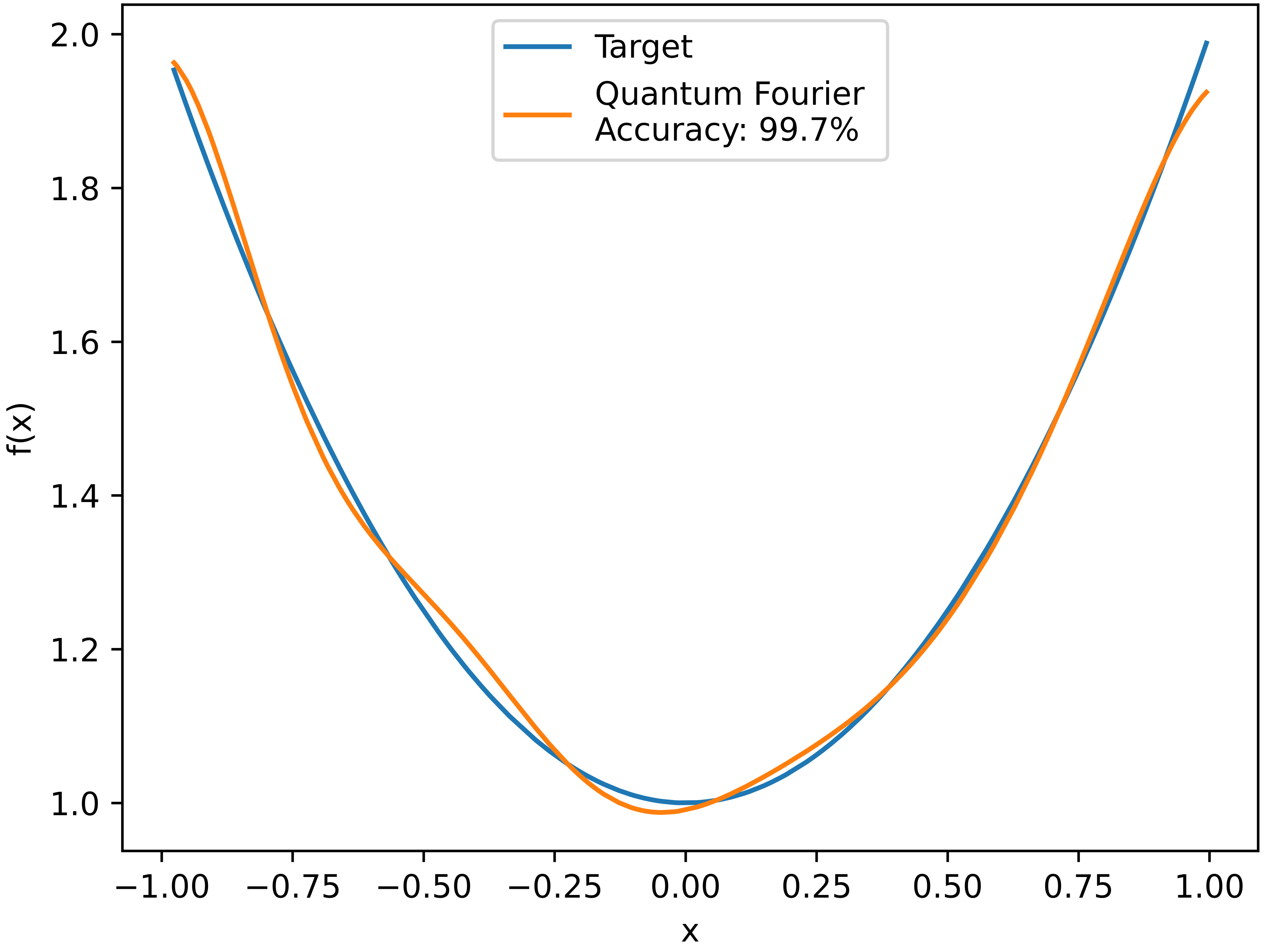
Speed up   No NISQ-friendly

QFIAE may achieve both

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 1. Fitting the function and obtaining the Fourier series



- Training the QNN with:

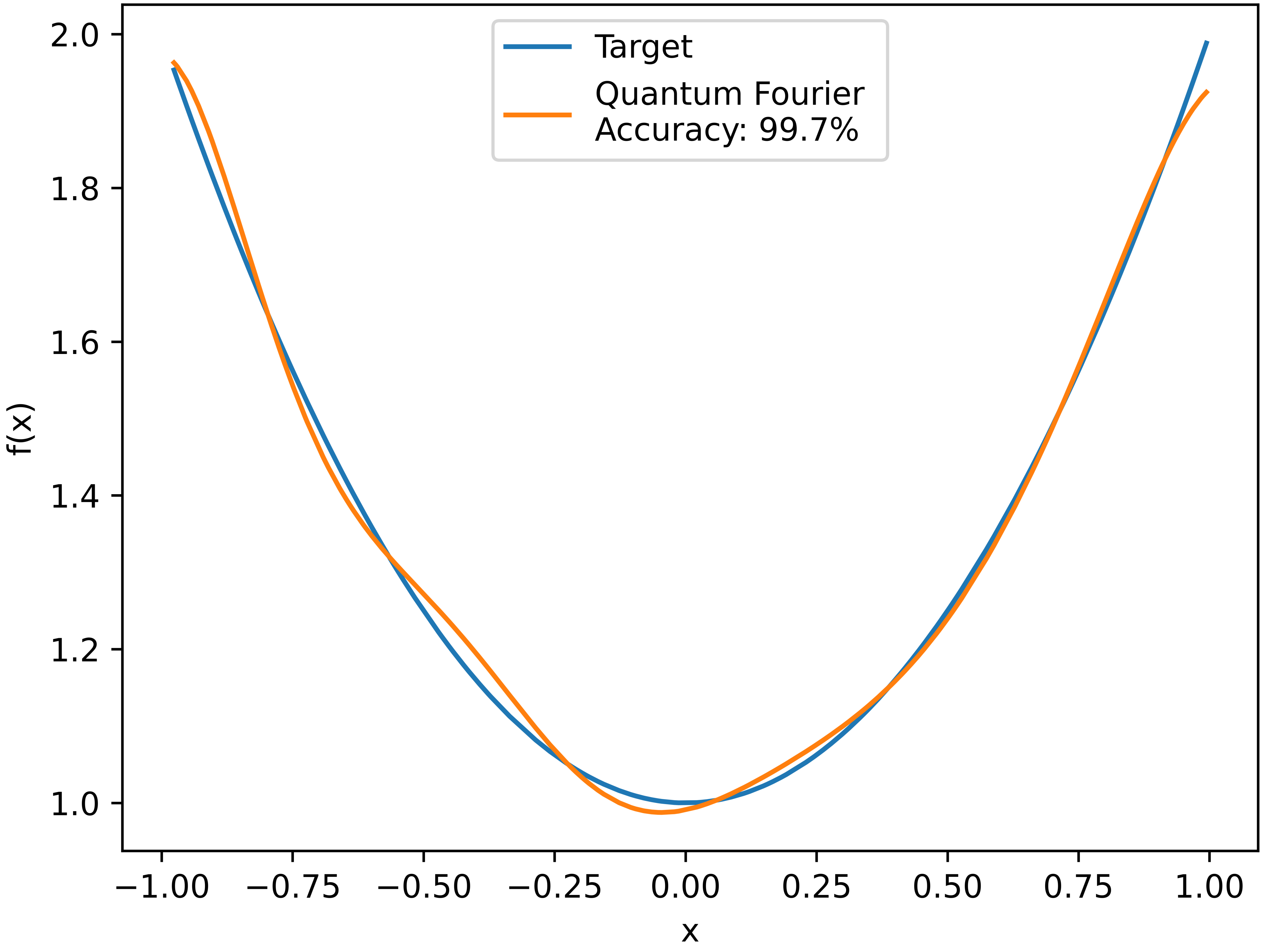
$$n_{qubits,QF} = 1 \quad L = 10 \quad \text{Adam Optimizer}$$

$$n_{data} = 200 \quad learning\_rate = 0.05 \quad nepochs = 100$$

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 1. Fitting the function and obtaining the Fourier series



- Training the QNN with:

$$n_{qubits,QF} = 1 \quad L = 10 \quad \text{Adam Optimizer}$$

$$n_{data} = 200 \quad learning\_rate = 0.05 \quad nepochs = 100$$

- Fourier series obtained:

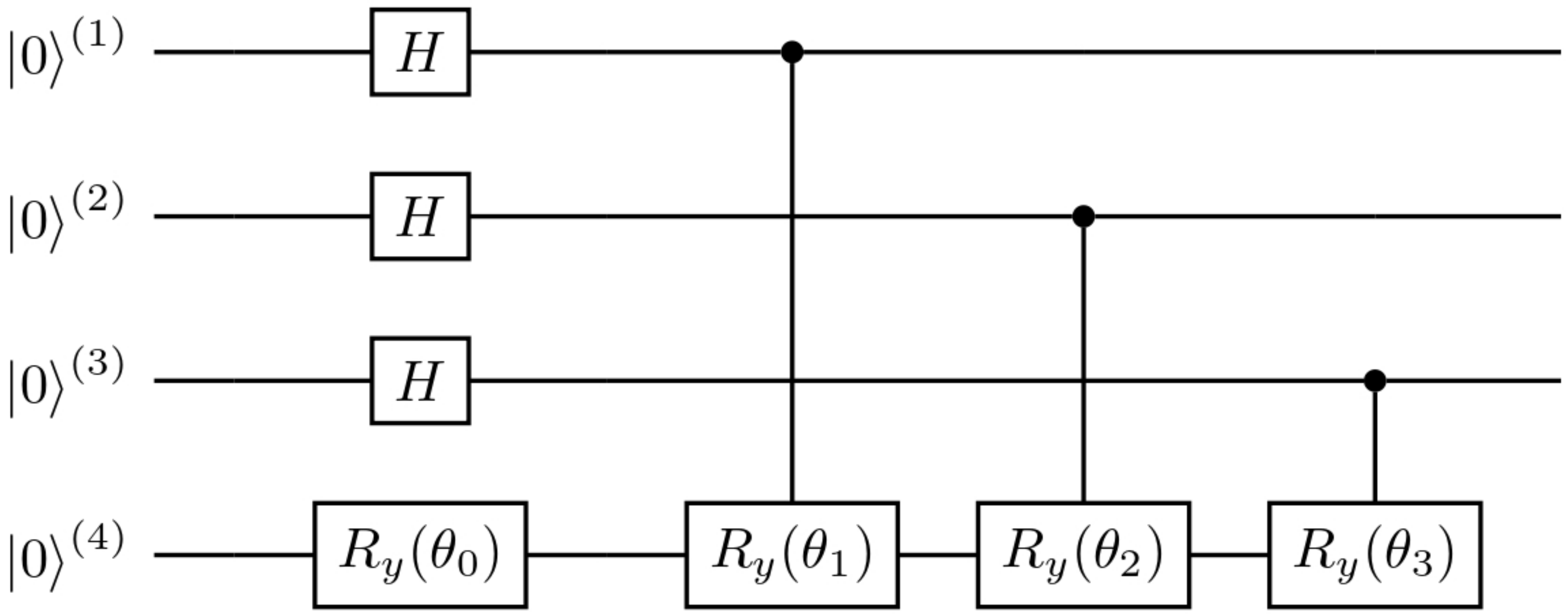
$$f(x) \approx 0.476 + 1.169\cos(x) - 0.263\cos(2x) + \dots$$
$$-0.017\cos(9x) + 0.004\cos(10x) +$$
$$-0.125\sin(x) - 0.278\sin(2x) + \dots$$
$$-0.029\sin(9x) - 0.004\sin(10x)$$

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 2. Applying IQAE to every trigonometric piece and calculating final integral

$\mathscr{A}$ circuit for $n_{qubits,IQAE} = 4$



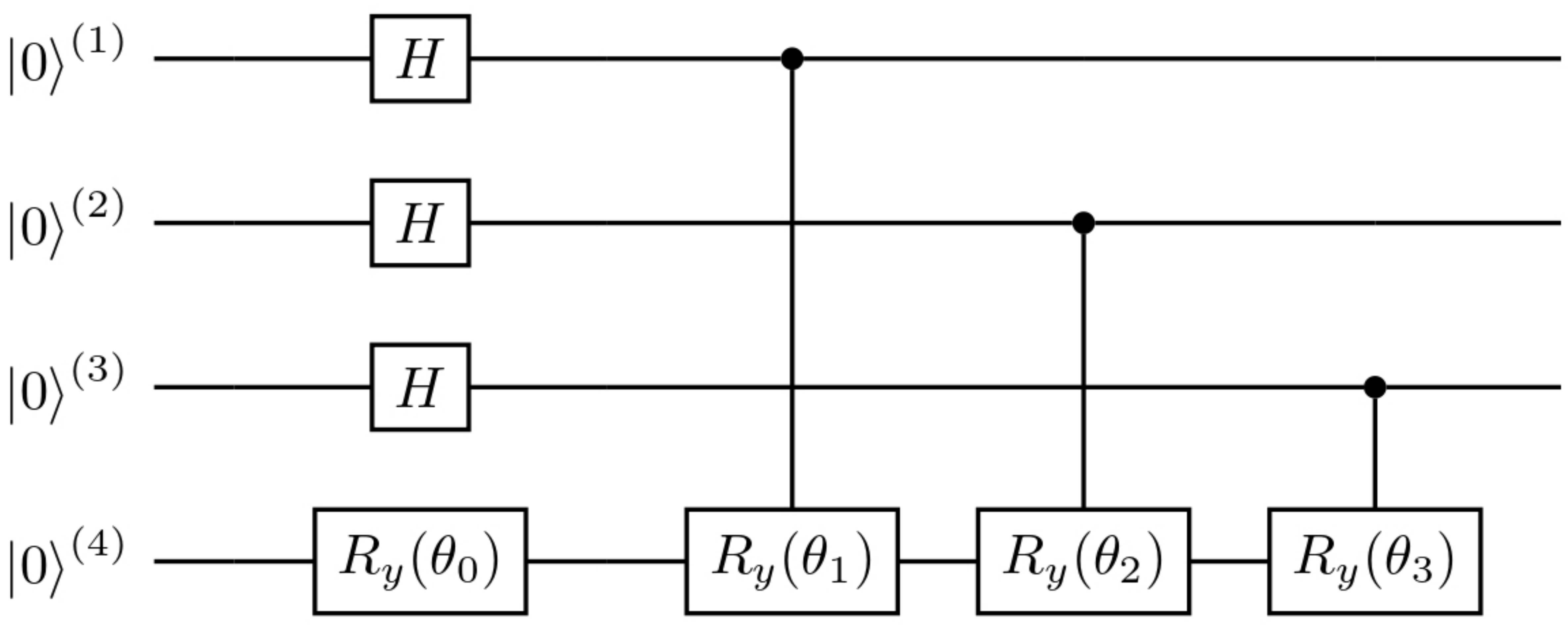
$\theta_0 = (x_{max} - x_{min})/2^n + 2x_{min}$ $\qquad$ $\theta_i = (x_{max} - x_{min})/2^{n-i-1}$

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

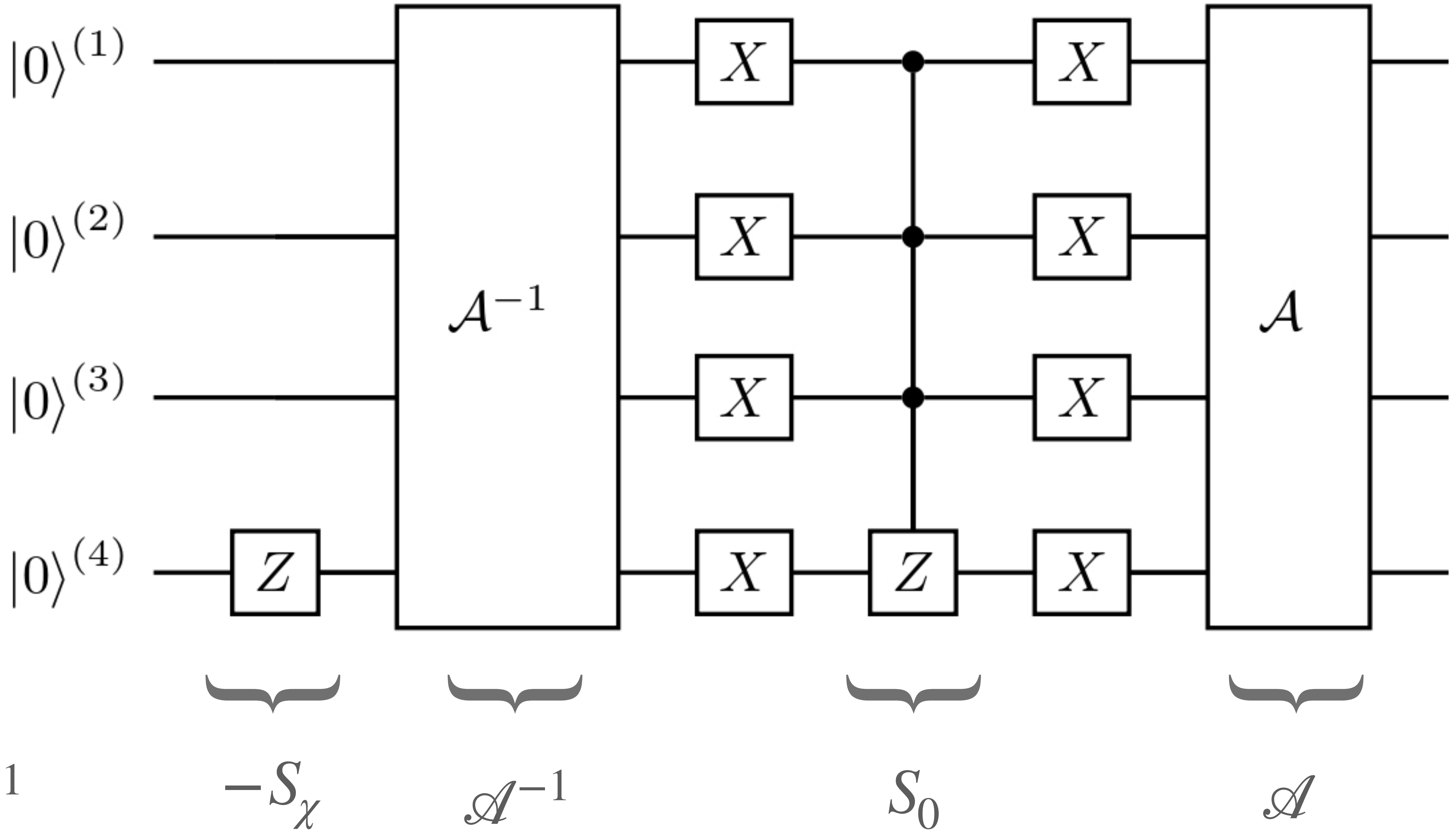
- 2. Applying IQAE to every trigonometric piece and calculating final integral

$\mathscr{A}$ circuit for $n_{qubits,IQAE} = 4$



$\theta_0 = (x_{max} - x_{min})/2^n + 2x_{min}$ $\theta_i = (x_{max} - x_{min})/2^{n-i-1}$

$\mathscr{Q}$ circuit for $n_{qubits,IQAE} = 4$



INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 2. Applying IQAE to every trigonometric piece and calculating final integral

method = QFIAE	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.32 ± 0.05	0.991
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.06	1.006
$n_{Fourier} = 5$, $shots = 1000$	1.33 ± 0.05	0.998
$n_{Fourier} = 10$, $shots = 1000$	1.33 ± 0.04	0.999
method = FQMCI	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.35 ± 0.07	1.010
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.07	1.003
$n_{Fourier} = 5$, $shots = 1000$	1.34 ± 0.07	1.007
$n_{Fourier} = 10$, $shots = 1000$	1.34 ± 0.06	1.002

Table 1: Integration of $1 + x^2$ from [0,1] with QFIAE and FQMCI

$$I = \int_0^1 dx(1 + x^2) = 4/3 \approx 1.333$$

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

➤ 2. Applying IQAE to every trigonometric piece and calculating final integral

method = QFIAE	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.32 ± 0.05	0.991
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.06	1.006
$n_{Fourier} = 5$, $shots = 1000$	1.33 ± 0.05	0.998
$n_{Fourier} = 10$, $shots = 1000$	1.33 ± 0.04	0.999
method = FQMCI	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.35 ± 0.07	1.010
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.07	1.003
$n_{Fourier} = 5$, $shots = 1000$	1.34 ± 0.07	1.007
$n_{Fourier} = 10$, $shots = 1000$	1.34 ± 0.06	1.002

Table 1: Integration of $1 + x^2$ from [0,1] with QFIAE and FQMCI

$$I = \int_0^1 dx(1 + x^2) = 4/3 \approx 1.333$$

Best results $n_{Fourier} = 10 \quad shots = 1000$

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 2. Applying IQAE to every trigonometric piece and calculating final integral

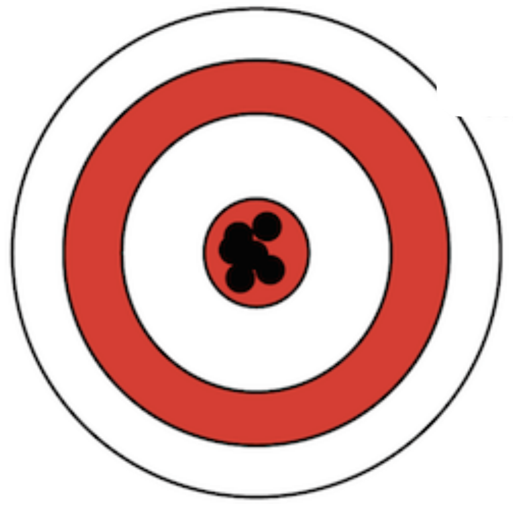
method = QFIAE	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.32 ± 0.05	0.991
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.06	1.006
$n_{Fourier} = 5$, $shots = 1000$	1.33 ± 0.05	0.998
$n_{Fourier} = 10$, $shots = 1000$	1.33 ± 0.04	0.999
method = FQMCI	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.35 ± 0.07	1.010
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.07	1.003
$n_{Fourier} = 5$, $shots = 1000$	1.34 ± 0.07	1.007
$n_{Fourier} = 10$, $shots = 1000$	1.34 ± 0.06	1.002

Table 1: Integration of $1 + x^2$ from [0,1] with QFIAE and FQMCI

$$I = \int_0^1 dx(1 + x^2) = 4/3 \approx 1.333$$

Best results $n_{Fourier} = 10 \quad shots = 1000$

- High accuracy $\epsilon \sim 0.1\,\%$
- High precision $\varepsilon \sim 2\,\%\,, \alpha = 0.05$



- May retain speed up

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 2. Applying IQAE to every trigonometric piece and calculating final integral

method = QFIAE	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.32 ± 0.05	0.991
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.06	1.006
$n_{Fourier} = 5$, $shots = 1000$	1.33 ± 0.05	0.998
$n_{Fourier} = 10$, $shots = 1000$	1.33 ± 0.04	0.999
method = FQMCI	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.35 ± 0.07	1.010
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.07	1.003
$n_{Fourier} = 5$, $shots = 1000$	1.34 ± 0.07	1.007
$n_{Fourier} = 10$, $shots = 1000$	1.34 ± 0.06	1.002

Table 1: Integration of $1 + x^2$ from [0,1] with QFIAE and FQMCI

$$I = \int_0^1 dx(1 + x^2) = 4/3 \approx 1.333$$

Best results $n_{Fourier} = 10 \quad shots = 1000$

- High accuracy $\epsilon \sim 0.1\,\%$
- High precision $\varepsilon \sim 2\,\%\,, \alpha = 0.05$

- May retain speed up

Is it NISQ-friendly?

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- 2. Applying IQAE to every trigonometric piece and calculating final integral

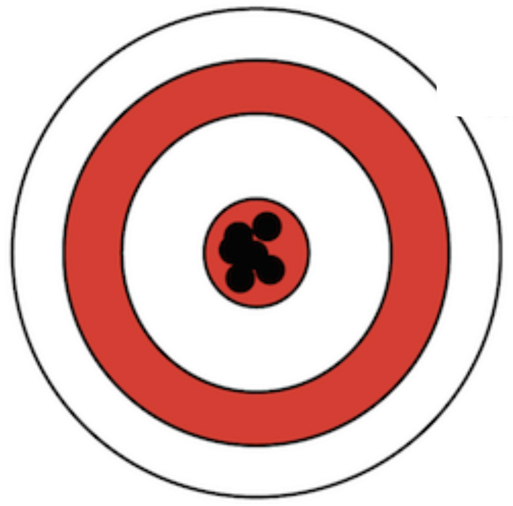
method = QFIAE	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.32 ± 0.05	0.991
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.06	1.006
$n_{Fourier} = 5$, $shots = 1000$	1.33 ± 0.05	0.998
$n_{Fourier} = 10$, $shots = 1000$	1.33 ± 0.04	0.999
method = FQMCI	I_{est}	ϵ
$n_{Fourier} = 5$, $shots = 100$	1.35 ± 0.07	1.010
$n_{Fourier} = 10$, $shots = 100$	1.34 ± 0.07	1.003
$n_{Fourier} = 5$, $shots = 1000$	1.34 ± 0.07	1.007
$n_{Fourier} = 10$, $shots = 1000$	1.34 ± 0.06	1.002

Table 1: Integration of $1 + x^2$ from [0,1] with QFIAE and FQMCI

$$I = \int_0^1 dx(1 + x^2) = 4/3 \approx 1.333$$

Best results $n_{Fourier} = 10 \quad shots = 1000$

- High accuracy $\epsilon \sim 0.1\,\%$
- High precision $\varepsilon \sim 2\,\%\,, \alpha = 0.05$



- May retain speed up

Is it NISQ-friendly?

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- Performing a depth analysis to check NISQ suitability

Depth analysis

$\text{QF}_{depth} = \mathcal{A}_{depth} + layers(\mathcal{A}_{depth} + \mathcal{S}_{depth})$			
$\mathcal{A}_{depth}$	$\mathcal{S}_{depth}$	$layers$	QF_{depth}
3	1	10	43
$\text{IQAE}_{depth} = \mathcal{A}_{depth} + k\mathcal{Q}_{depth}$			
$\mathcal{A}_{depth}$	$\mathcal{Q}_{depth}$	k	IQAE_{depth}
4	12	9	112

Table 2: Depths of the Quantum Fourier (QF) and Iterative Quantum Amplitude Estimation (IQAE) parts of the QFIAE method.

INTEGRATION OF A PARTICLE PHYSICS PROCESS

QFIAE JML, Grossi, Cieri, Rodrigo, 2305.01686

- Performing a depth analysis to check NISQ suitability

Depth analysis

$\text{QF}_{depth} = \mathcal{A}_{depth} + layers(\mathcal{A}_{depth} + \mathcal{S}_{depth})$			
$\mathcal{A}_{depth}$	$\mathcal{S}_{depth}$	$layers$	QF_{depth}
3	1	10	43
$\text{IQAE}_{depth} = \mathcal{A}_{depth} + k\mathcal{Q}_{depth}$			
$\mathcal{A}_{depth}$	$\mathcal{Q}_{depth}$	k	IQAE_{depth}
4	12	9	112

Table 2: Depths of the Quantum Fourier (QF) and Iterative Quantum Amplitude Estimation (IQAE) parts of the QFIAE method.

High probability of success in hardware

IONQ **QUANTINUUM**

IonQ, "Algorithmic qubits: A better single-number metric." https://ionq.com/resources/algorithmic-qubits-a-better-single-number-metric, 2023.

TUTORIAL

QFIAE JML, Grossi, Cieri, Rodrigo, [2305.01686](2305.01686)

Open-source framework for quantum computing

https://qibo.science

- ➤ Change f(x)
- ➤ Change p(x)
- ➤ Change QNN
- ➤ Change number of qubits
- ➤ Much more!

https://qibo.science/qibo/stable/code-examples/tutorials/qfiae/qfiae_demo.html

CONCLUSIONS

What is QFIAE? 🤔

- In a nutshell: QFIAE = QML+QAE → Quantum Monte Carlo Integrator

CONCLUSIONS

What is QFIAE?

- In a nutshell: QFIAE = QML+QAE → Quantum Monte Carlo Integrator

Where is the novelty?

- Based on FQMCI, but using a QNN to obtain the Fourier series

CONCLUSIONS

What is QFIAE?

- In a nutshell: QFIAE = QML+QAE → Quantum Monte Carlo Integrator

Where is the novelty?

- Based on FQMCI, but using a QNN to obtain the Fourier series

Where is the advantage?

- QFIAE keeps QAE (Grover's) advantage → quadratic speedup querying $p(x)$

CONCLUSIONS

What is QFIAE?

- In a nutshell: QFIAE = QML+QAE ➡ Quantum Monte Carlo Integrator

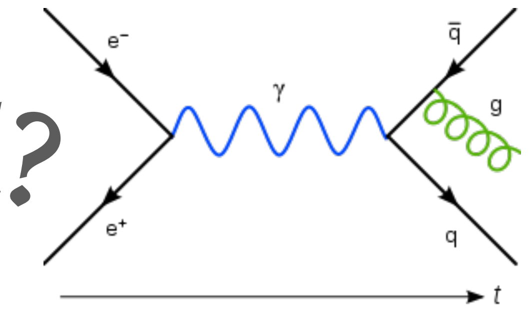
Where is the novelty?

- Based on FQMCI, but using a QNN to obtain the Fourier series

Where is the advantage?

- QFIAE keeps QAE (Grover's) advantage ➡ quadratic speedup querying $p(x)$

Where has it been applied?

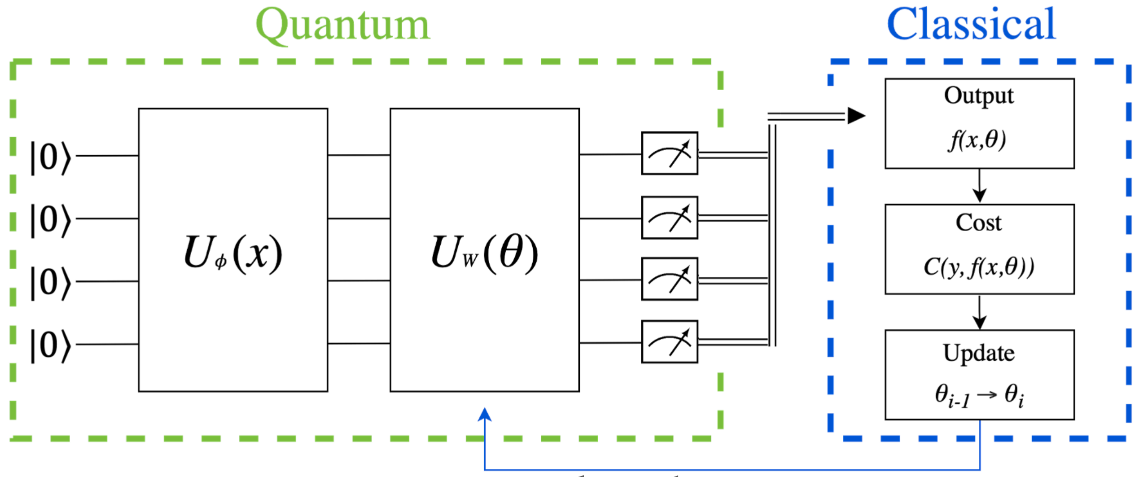


- Elementary particle physics process ➡ very accurate results and NISQ-friendly!

FUTURE WORK

- Apply the method to **non-smooth** functions $f(x)$
  - Extend the method to N-**dimensional** functions $f(\vec{x})$
  - Integrate higher multiplicity particle physics processes

Work in progress → **Loop Feynman integrals**

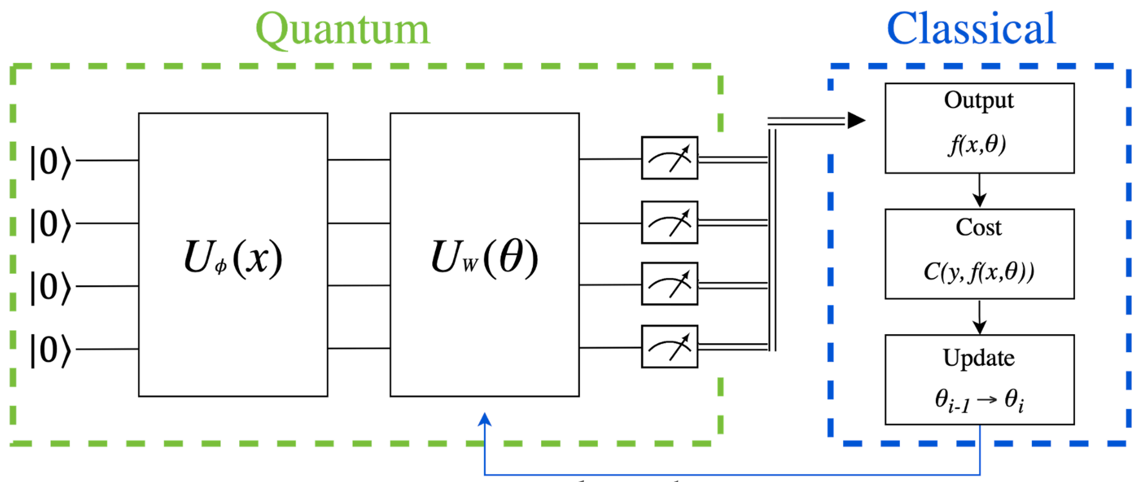


Test QNN expressibility

FUTURE WORK

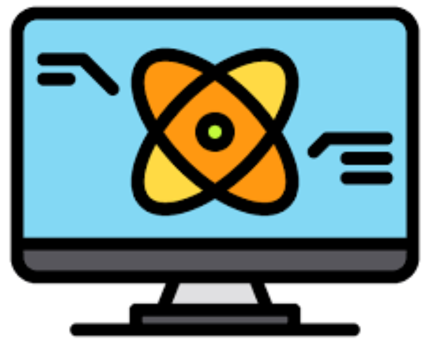
- Apply the method to **non-smooth** functions $f(x)$
  - Extend the method to N-**dimensional** functions $f(\vec{x})$
  - Integrate higher multiplicity particle physics processes

Work in progress → **Loop Feynman integrals**



Test QNN expressibility

- Apply the method to **non-planar probability distributions** $p(x)$
  - Actually obtain a **speedup** over classical MC
  - Finding a **suitable** $p(x)$ encodable into a **shallow** circuit

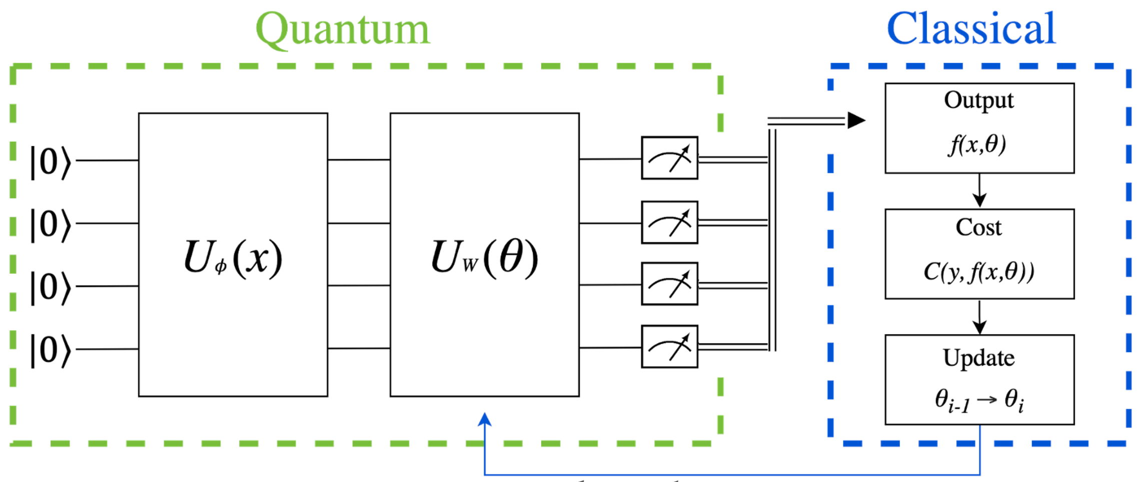


Test NISQ-friendliness

FUTURE WORK

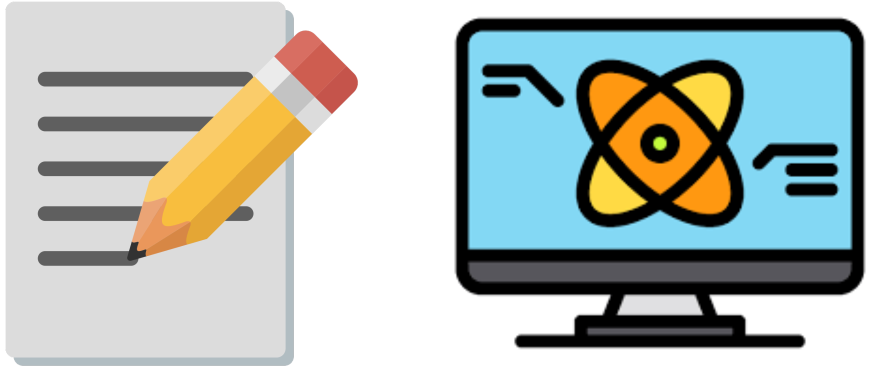
- Apply the method to **non-smooth** functions $f(x)$
  - Extend the method to N-**dimensional** functions $f(\vec{x})$
  - Integrate higher multiplicity particle physics processes

Work in progress → Loop Feynman integrals



Test QNN expressibility

- Apply the method to **non-planar probability distributions** $p(x)$
  - Actually obtain a **speedup** over classical MC
  - Finding a **suitable** $p(x)$ encodable into a **shallow** circuit

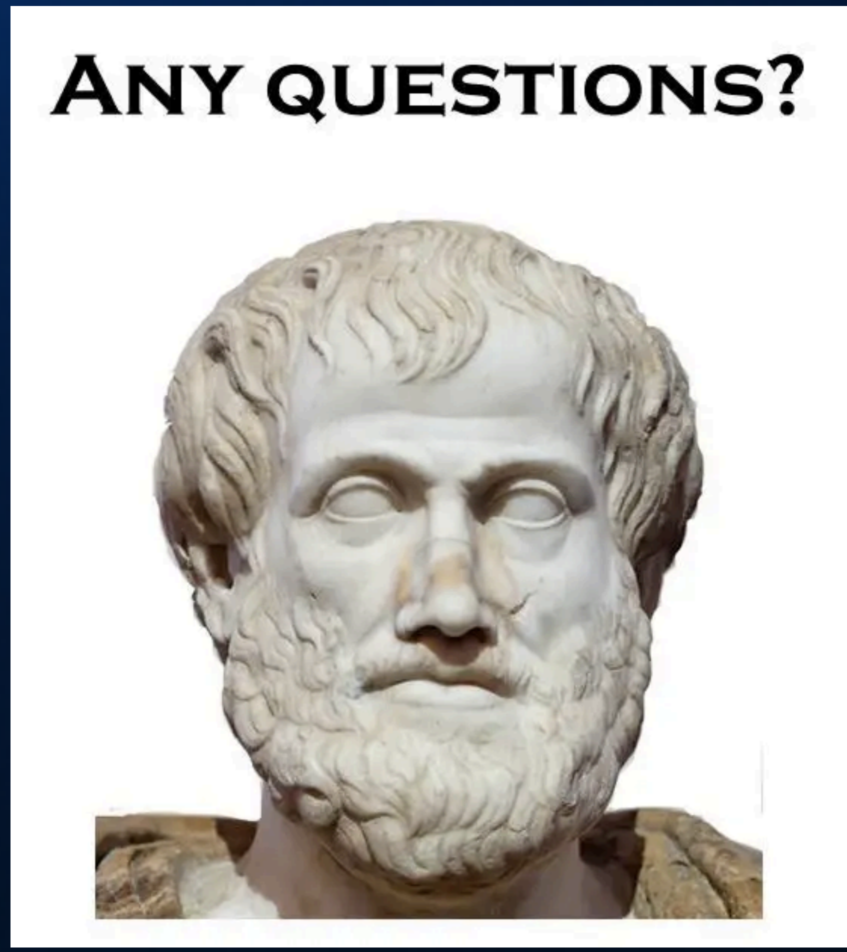


Test NISQ-friendliness

- Deployment on **hardware** and benchmarking with **other integration methods**

Test practical utility

THANK YOU FOR YOUR ATTENTION!!



Link to the paper!

arXiv:2305.01686

Link to the tutorial!

https://qibo.science/qibo/stable/code-examples/tutorials/qfiae/qfiae_demo.html

Contact me:

Jorge.M.Lejarza@ific.uv.es

Gorka Martínez de Lejarza Samper

@gmlejarza

BACK UP

QUANTUM MONTE CARLO: EXAMPLE $\sin(x)^2$

- Let's consider a simple trigonometric function: $\sin(x)^2$

$$I = \frac{1}{x_{max} - x_{min}} \int_{x_{min}}^{x_{max}} \sin(x)^2 dx$$

- Discretizing the integral in n-qubit and considering $p(x) = 1/2^n$

$$\mathscr{P} |0\rangle_n |0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle_n |0\rangle$$

$$\mathscr{R} |x\rangle_n |0\rangle = |x\rangle_n \left(\sin\left(\frac{(x+1/2)(x_{max} - x_{min})}{2^n} \right) |1\rangle + \cos\left(\frac{(x+1/2)(x_{max} - x_{min})}{2^n} \right) |0\rangle \right)$$

INTEGRATION OF A PARTICLE PHYSICS PROCESS: DETAILED CALCULATION

- Let's consider:

$$f(x) = 1 + x^2;\ p(x) = 1/2^n;\ x_{max} = 1;\ x_{min} = 0;\ n_{Fourier} = 5$$

- Then the integral is:

$$I = \frac{1}{x_{max} - x_{min}} \int_{x_{min}}^{x_{max}} p(x) f(x) dx = \int_0^1 (1 + x^2) dx = 4/3 \approx 1.333$$

- So, the results will be:

$$I_{estimated} = \int_0^1 [\underbrace{\left(1 + \frac{\pi^2}{3}\right)}_{\sim 4.290} + \underbrace{(-4\cos x)}_{\substack{\textbf{IQAE} \\ \sim -3.366}} + \underbrace{(\cos 2x)}_{\substack{\textbf{IQAE} \\ \sim 0.455}} + \underbrace{\left(-\frac{4}{9}\cos 3x\right)}_{\substack{\textbf{IQAE} \\ \sim -0.021}} + \underbrace{\left(\frac{1}{4}\cos x\right)}_{\substack{\textbf{IQAE} \\ \sim -0.047}} + \underbrace{\left(-\frac{4}{25}\cos 5x\right)}_{\substack{\textbf{IQAE} \\ \sim 0.031}}] dx \approx 1.341$$