Classical Simulability of Quantum Circuits

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QUANTUM
TECHNOLOGY
INITIATIVE

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Quantum Computing

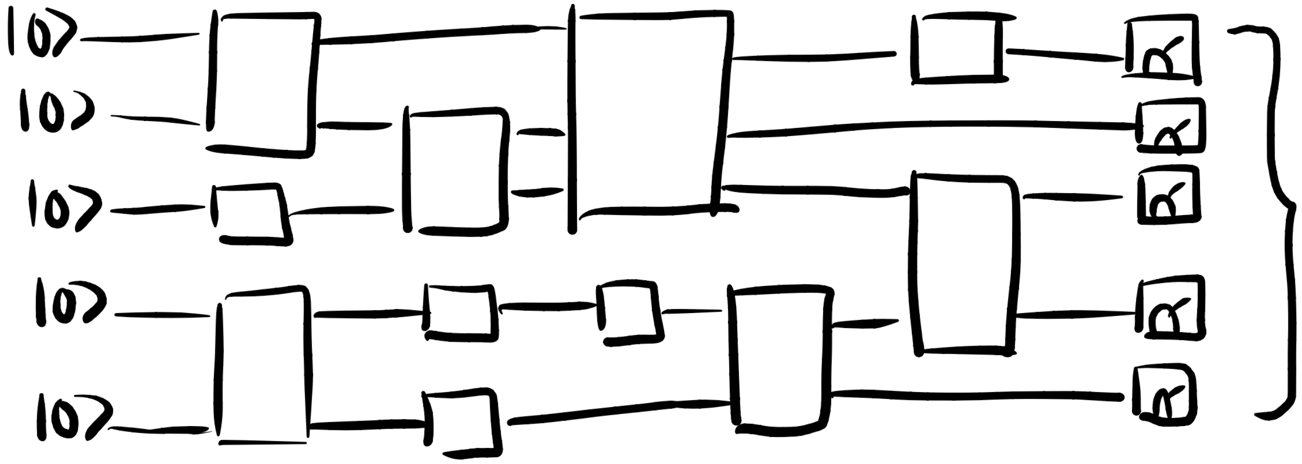
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

superposition

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

entauglement

Quantum Circuits and *the Born rule*



An arbitrary quantum circuit generating the state $|\Psi\rangle$

Quantum Measurement

→ retrieve a classical output distribution $|\langle x|\Psi\rangle|^2$

(with $x \in \{0,1\}^n$) according to Born rule

What do we mean by ***efficient classical simulability***?

strong sense

weak sense

Compute probabilities of the **output measurement efficiently classically** with high accuracy

Sample from the output distribution **efficiently** using a classical computer

Jozsa, Richard, and Maarten Van den Nest. "Classical simulation complexity of extended Clifford circuits." arXiv preprint arXiv:1305.6190 (2013).

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Computation in polynomial time
(*poly(N)* with *N* the number of operations)

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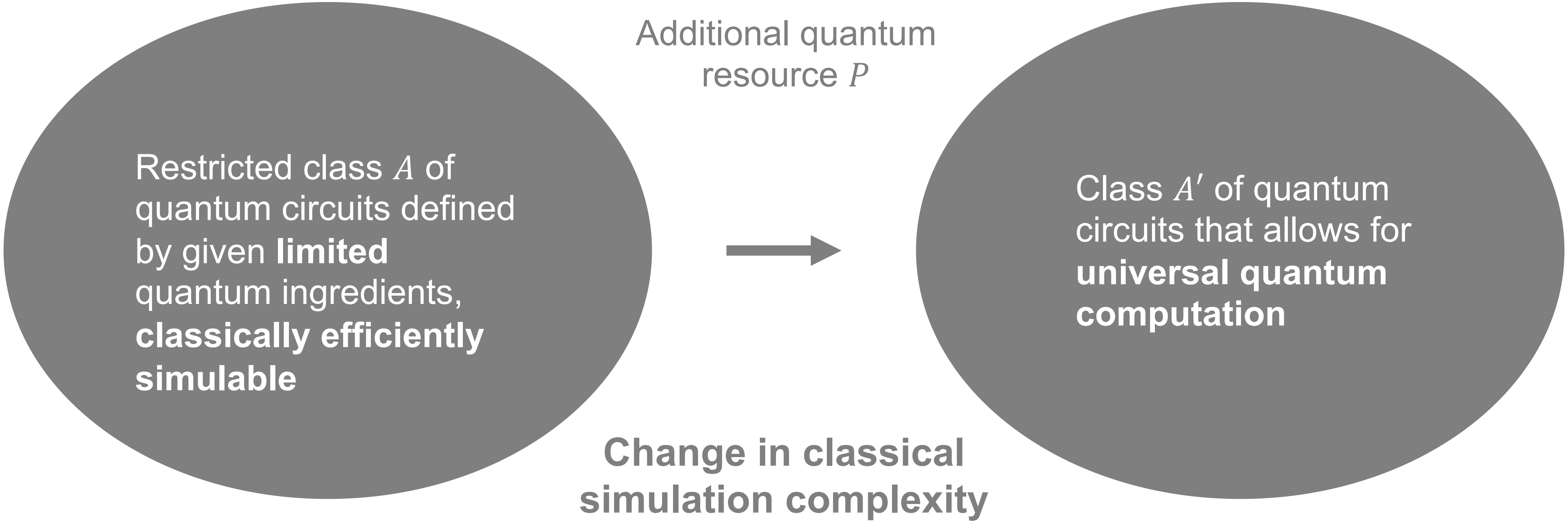
Considering gate sets in the circuit model

What is the fundamental reason for quantum speed-up?



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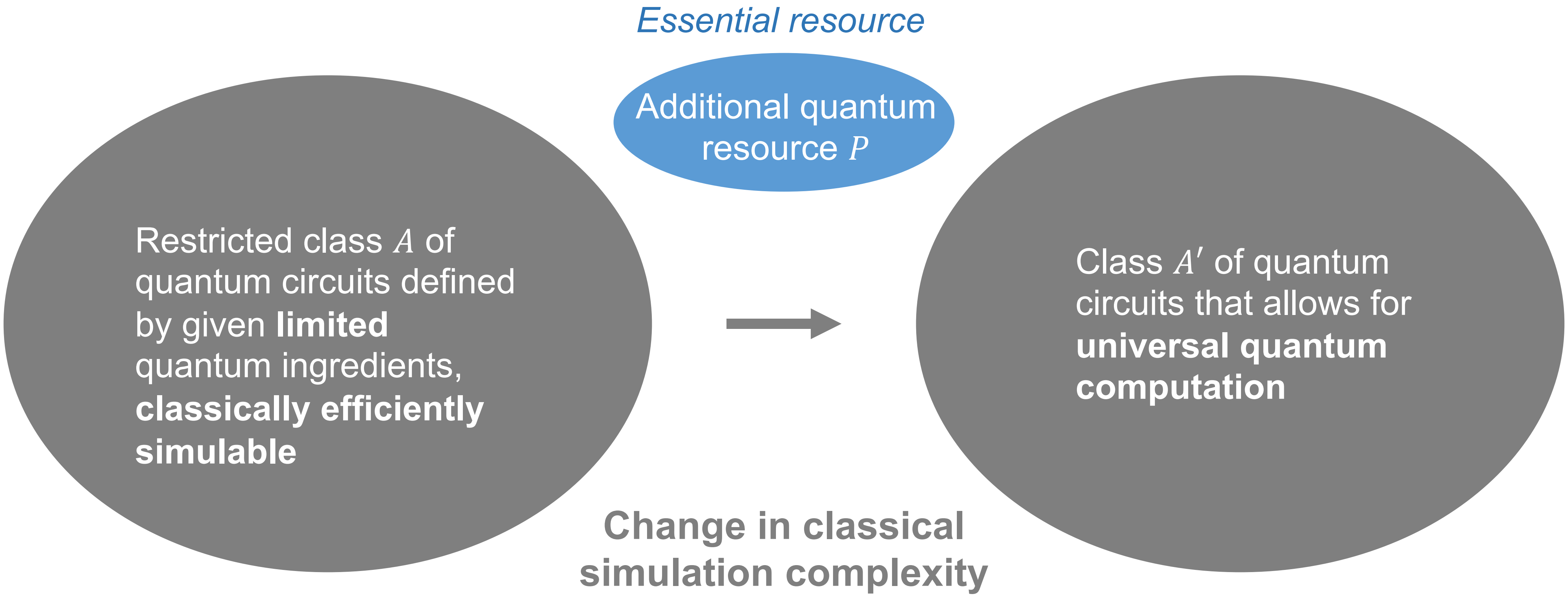

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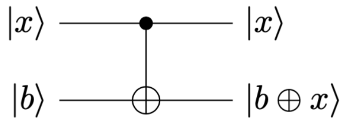
Universal gate sets – *circuit model*

$$\{\text{AND}, \text{NOT}, \text{OR}\}$$

$$\{H, S, T, CNOT\}$$

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



Universal for **classical** computation

→ Universal logic gate set that can be used to compute an arbitrary classical function

Universal for **quantum** computation

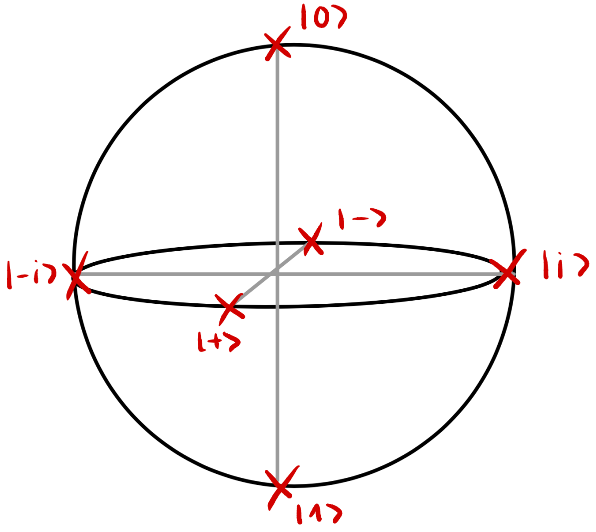
→ Any unitary can be approximated to arbitrary accuracy by a quantum circuit including only gates of this set

Nielsen, Michael A., and Isaac Chuang. "Quantum computation and quantum information." (2002).

The Clifford group – *an efficiently classically simulable group*

The Clifford group $\mathcal{C}_n = \{V \in U_{2^n} | V \mathcal{P}_n V^\dagger = \mathcal{P}_n\}$ is the normalizer of the Pauli group $\mathcal{P}_n = \left\{ e^{i\frac{\theta}{2}} \sigma_{j_1} \otimes \ldots \otimes \sigma_{j_n} \middle| \, \theta = 0,1,2,3; j_k = 0,1,2,3 \right\}$.

Pauli matrices



Bloch sphere: only the marked points are produced by the Clifford operators

$$\{H, S, \cancel{T}, CNOT\}$$

Set of generators of the Clifford group
(Clifford gates are the elements of the Clifford group)

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Pauli matrices

Essential resource to form universal gate set,
a non-Clifford gate, e.g., $T = diag(1, e^{i\pi/4})$

$$\{H, S, \not{T}, CNOT\}$$

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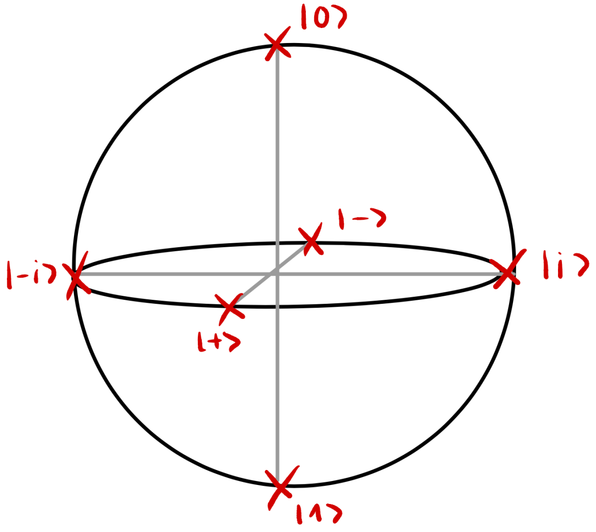


Bloch sphere: only the marked points are produced by the Clifford operators

Efficient classical simulation of Clifford gates

The Gottesman-Knill theorem

A quantum circuit build up of Clifford gates can be ***efficiently simulated*** *on a* ***classical computer****.*
(Qubit preparation and measurement in computational basis.)

There are more detailed considerations of cases with different computational complexities.

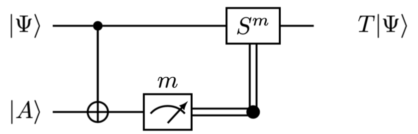
→ Even **highly entangled states** can be efficiently classically simulated.

Nielsen, Michael A., and Isaac Chuang. "Quantum computation and quantum information." (2002).
Gottesman, Daniel. "The Heisenberg representation of quantum computers." *arXiv preprint quant-ph/9807006* (1998).

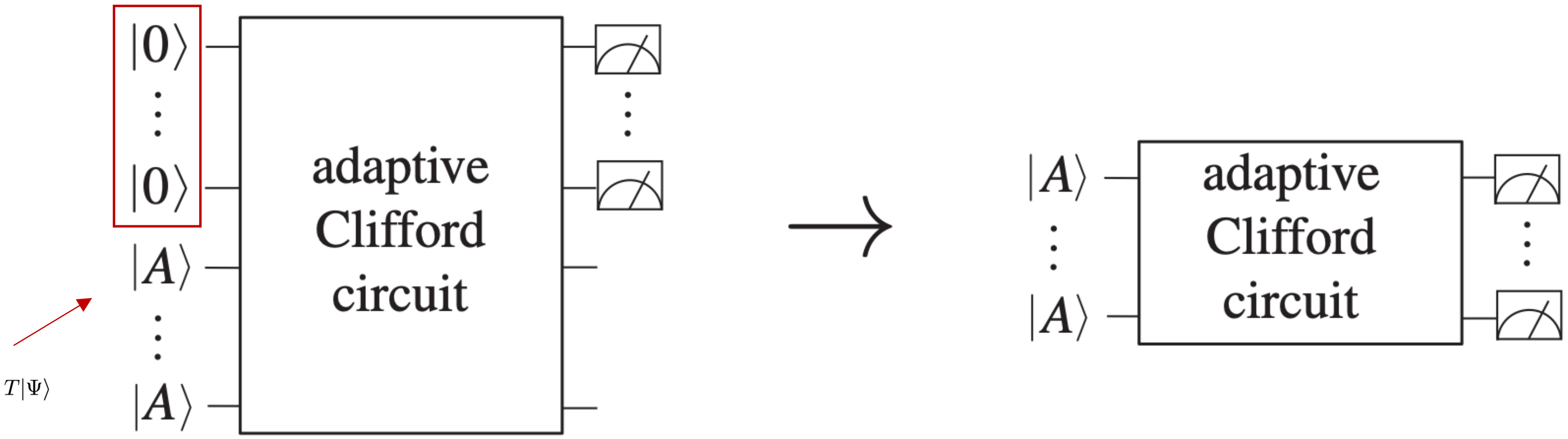
Generating set of the Clifford group: $\langle \boldsymbol{H}, \boldsymbol{S}, \boldsymbol{CNOT} \rangle$

Eliminate parts of the quantum circuit through classical computation

An extension of the Gottesman-Knill theorem



T-gadget



A universal quantum circuit with T-gates expressed as T-gadgets can be **compressed with a polynomial overhead**. The compression **removes all stabilizer inputs.**

→ **Reduce the number of qubits.**

Yoganathan, Mithuna, Richard Jozsa, and Sergii Strelchuk. "Quantum advantage of unitary Clifford circuits with magic state inputs." Proceedings of the Royal Society A 475.2225 (2019): 20180427.

$$|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{\frac{i\pi}{4}}|1\rangle)$$

Classical simulability – *Outlook*

- Consider different non-universal and classically simulable gate sets and their **extension to universality**
- How and which quantum circuits can we **efficiently classically compress** and thus be resource efficient?
- Focus on quantum circuits that are **hard to simulate classically** in order to expect a speed-up over classical computation
- Apply results on quantum circuit architectures for **encoding** (classical) data and parametrized **variational form**

Thank you!

Are there any questions?

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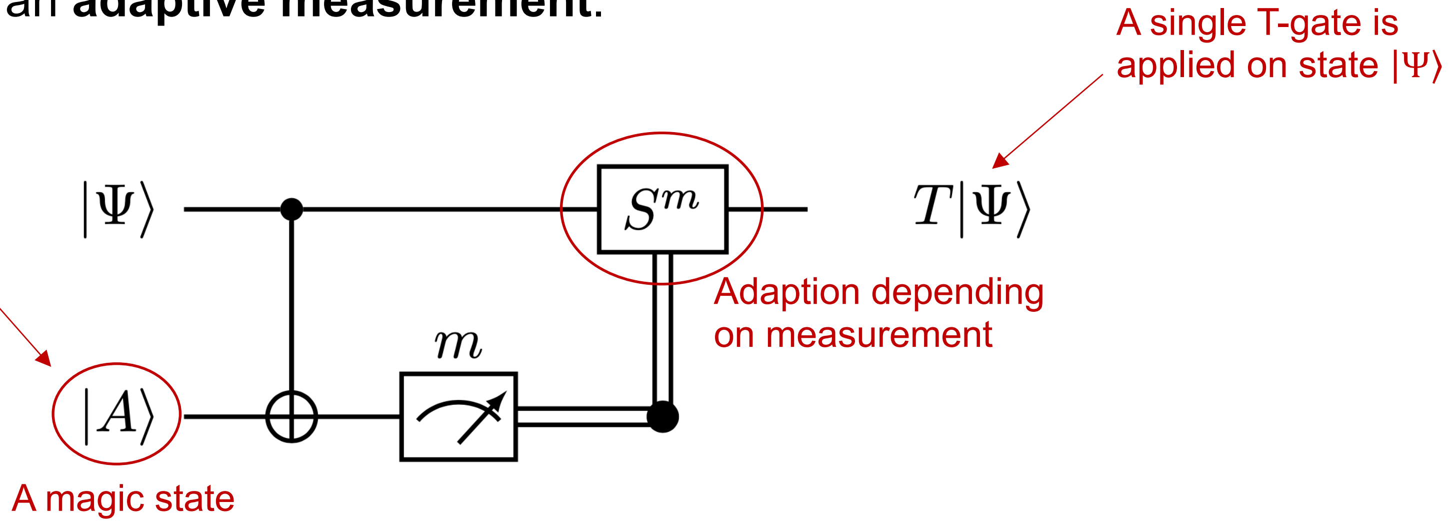
Sofia Vallecorsa, Martin Werner

Backup

Towards an extensions of the Gottesman-Knill theorem

Recap: T-gate ($T = diag(1, e^{i\pi/4})$) extends the Clifford group to a universal set.

A single T-gate can be implemented by a **T-gadget** using one **magic state** $|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{\frac{i\pi}{4}}|1\rangle)$ using an **adaptive measurement**.



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